

Measuring efficiency in retail planning



Kurt Marais

Thesis presented in partial fulfilment of the requirements for the degree of
Master of Commerce (Operations Research)
in the Faculty of Economic and Management Sciences at Stellenbosch University

Declaration

By submitting this thesis electronically, I declare that the entirety of the work contained therein is my own, original work, that I am the sole author thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

Date: April 2019

Abstract

Efficiency is the measure of how well a process performs, and businesses are constantly looking for ways to improve their productivity. Traditional performance measures are commonly used and applied to data, but often do not consider the effect that multiple inputs and outputs have on the performance of a service unit. Thus, it is important to measure efficiency within the current capabilities of service units. One way to measure the capabilities of efficiency is through benchmarking, which identifies best-practice service units and compares all service units to the best practices. The benchmarking tool used in this study that embodies this notion is known as data envelopment analysis. Data envelopment analysis (DEA) is a linear programming tool used to determine relative efficiency for a group of service units and provides a score on the level of efficiency relative to other service units.

DEA is applied to the data of a prominent South African retailer, and multiple DEA models are applied to the data to provide insight into the efficiency of service units for the considered retailer. Numerous extensions and adaptations of DEA have been developed to provide deeper insights into the efficiency of service units, depending on the available data. The CCR model and the BCC model are the main DEA models used in this thesis. Multiple regression analysis is also performed on the efficiency scores of DEA and the information that the models require. Important components for DEA are the decision of inputs and outputs, as well as the number of service units considered at one time, all of which have an effect on the discriminatory power of the models. The data are grouped into categories and DEA is run on these groups to better understand the results that DEA provides. The efficiency scores from the different models are determined for each of the considered service units order for the retailer to make decisions on minimising resources or maximising its outputs in future. DEA is not only a diagnostic tool for determining where inefficiencies exist, but how these inefficiencies should be approached, relative to best-practice units.

DEA results were applied to data of 1 207 stores over 26 weeks, and it was identified that new fashion products generally perform better than older products. Regression analysis used for productivity measurement, while better for statistical analysis when compared to DEA, is limited in its ability to calculate efficiency for multiple inputs and multiple outputs at once. The results also provide confirmation on the discriminatory power of the choice of components used in DEA, and that isolating one component as a measure of efficiency is not enough for service units, since performance is dependent on multiple factors. The overall result is that DEA be used in tandem with other performance measures to diagnose where inefficiencies occur, and use the information of DEA to move towards improved productivity.

Uittreksel

Doeltreffendheid is die mate van hoe goed 'n proses verrig word, en besighede soek voortdurend maniere om hul produktiwiteit te verbeter. Tradisionele prestasiemaatsawwe word algemeen gebruik en toegepas op data, maar beskou dikwels nie die effek wat verskeie insette en uitsetter op die prestasie van 'n dienseenheid het nie. Dit is dus belangrik om doeltreffendheid binne die huidige vermoëns van dienseenhede te meet. Een manier om die vermoëns van doeltreffendheid te meet, is deur middel van maatstafmetodes, wat beste dienseenhede identifiseer en all dienseenhede vergelyk met die beste pratyke. Die maatstafmetode wat gebruik word in hierdie studie, staan bekend as data-omhullingsanalise. Data-omhullingsanalise (DEA) is 'n lineêre programmeringsinstrument wat gebruik word om relatiewe doeltreffendheid vir 'n groep dienseenhede te bepaal en bied 'n telling op die vlak van doeltreffendheid relatief tot ander dienseenhede.

DEA word toegepas op die data van 'n prominente Suid-Afrikaanse kleinhandelaar en verskeie DEA-modelle word op die data toegepas om insig te gee in die doeltreffendheid van dienseenhede vir hierdie kleinhandelaar. Verskeie uitbreidings en aanpassings van DEA is ontwikkel om die doeltreffendheid van dienseenhede beter te verstaan, afhangende van die beskikbare data. Die CCR-model en die BCC-model is die hoof DEA-modelle wat in hierdie studie gebruik word. Meervoudige lineêre regressie analise word ook uitgevoer op die tellings en die inligting wat die modelle benodig. Belangrike komponente vir DEA is die besluit van insette en uitsette, sowel as die aantal dienseenhede wat op 'n slag oorweeg word. Hierdie komponente het 'n uitwerking op die diskriminerende krag van die modelle. Die data word in kategorieë gegroepeer en DEA word op hierdie groepe uitgevoer om die resultate beter te verstaan. Die tellings van die verskillende modelle word bepaal vir elkeen van die oorweegde dienseenhede sodat die handelaar besluite kan neem oor die vermindering van hulpbronne of die maksimering van sy uitsette in die toekoms. DEA is nie net 'n diagnostiese hulpmiddel om te bepaal waar ondoeltreffendheid bestaan nie, maar ook hoe om hierdie ondoeltreffendheid te benader, in vergelyking met doeltreffende dienseenhede.

DEA resultate is toegepas op data van 1 207 winkels oor 26 weke, en dit is bepaal dat nuwe modeprodukte oor die algemeen beter presteer as ouer produkte. Regressie-analise wat gebruik word vir produktiwiteitsmeting is beperk in die vermoë om effektiwiteit vir verskeie insette en veelvoudige uitsette gelyktydig te bereken, alhoewel dit beter is vir statistiese analise in vergelyking met DEA. Die resultate bied ook bevestiging van die diskriminerende krag van die keuse van komponente wat in die DEA gebruik word, en dat all komponente as 'n mate van doeltreffendheid beskou moet word, aangesien prestasie afhanklik is van die verskeie komponente. Die algehele resultaat is dat DEA saam met ander prestasiemaatsawwe gebruik word om ondoeltreffendheid te identifiseer, en om die inligting van DEA te gebruik om produktiwiteit te verbeter.

Acknowledgements

Many people and institutions played a significant role in the work leading up to and during the writing of this thesis. The author wishes to express his deepest gratitude towards:

- Prof SE Visagie for his enthusiasm, support, guidance, and insight through the years as my supervisor.
- The Department of Logistics at Stellenbosch University for providing technical and professional support, not only contributing towards the completion of this thesis, but also towards my personal development.
- My parents, Randall and Elsabeth, and to my brother, Ryan, for their constant support, motivational messages and belief in my abilities throughout my academic career.
- The students from the Postgrad Lab, Chesme Messina, Flora Hofmann, Gavin le Roux, Bryce Senekal, Charl van Rooyen, Noé Fouotsa Manfou and Kyle van Heerden, for providing additional support, a sense of solidarity and relief from work every now and then.
- The IDEE research groups, whom have contributed invaluable support and advice.
- My friends from various walks of life, be it from leadership spaces on campus, or my fellow choristers from the Stellenbosch University Choir, for all of your genuine expressions of interest, and your friendship.
- The generous financial support from PEP, without whom it would not have been possible to complete this thesis.

Table of Contents

List of Reserved Symbols	xiii
List of Acronyms	xv
List of Figures	xvii
List of Tables	xix
1 Introduction	1
1.1 The retail supply chain	1
1.2 The distribution network of a retail chain	2
1.3 Stages of planning	3
1.4 Productivity or efficiency	5
1.5 Benchmarking	6
1.6 Data envelopment analysis	6
1.7 Problem description	7
1.8 Scope of thesis	8
1.9 Thesis objectives	8
1.10 Thesis structure	9
2 Literature review	11
2.1 Productivity and benchmarking measures	12
2.1.1 Regression analysis	12
2.1.2 Stochastic frontier analysis	15
2.1.3 Goal programming	15
2.2 Extensions of DEA	16
2.2.1 The CCR ratio model	16
2.2.2 The BCC model	16
2.2.3 The multiplicative DEA models	17
2.2.4 The additive DEA models	17

2.2.5	Slack-based measure of efficiency	17
2.2.6	Super-efficiency and cross-efficiency	18
2.3	Applications of DEA	18
2.4	Efficiency measurement in the retail industry	19
2.5	Input and output mix	21
2.6	The benefits of DEA	22
2.7	The shortcomings of DEA	22
3	DEA models	25
3.1	The mathematical formulation of DEA	25
3.2	DEA efficient and weak efficient units	27
3.3	Constant returns to scale	28
3.3.1	Input-oriented frontier of CRS	30
3.3.2	Output-oriented frontier of CRS	32
3.4	Variable returns to scale	34
3.4.1	Input-oriented frontier of VRS	36
3.4.2	Output-oriented frontier of VRS	38
3.5	Technical and scale efficiency	40
4	Data validation and analysis	43
4.1	Introduction to the dataset	44
4.1.1	Data attributes	44
4.1.2	Store attributes	45
4.1.3	The dataset	46
4.2	Data validation	47
4.2.1	Data entries	48
4.2.2	Duplicates of data entries	48
4.2.3	Negative entries	48
4.2.4	Disclosure of weeks	48
4.2.5	Uniqueness of store names	48
4.2.6	Flow of inventory	49
4.3	Data analysis	49
4.3.1	Base exclusive values	49
4.3.2	Opening and closing stock	49
4.3.3	Inflow quantity	50
4.3.4	Sales	50

4.3.5	Style code	51
4.3.6	Stores sales plan value	52
4.3.7	Service level value	52
4.3.8	Last day of the week (LDOW)	52
5	DEA components	53
5.1	Grouping criteria	54
5.2	Outputs	54
5.2.1	Rate of sales	55
5.2.2	Turnover	55
5.2.3	Store service level	56
5.3	Inputs	56
5.3.1	Width	56
5.3.2	Inflow quantity	57
5.3.3	Full price percentage	57
5.4	Assumptions on the models	57
5.5	The DEA models	58
6	Results	59
6.1	DEA for store DMUs	59
6.1.1	Performance of store formats	60
6.1.2	Stores with products of season W17	63
6.1.3	Stores with products of season W16	66
6.1.4	Stores with replenishment products	68
6.2	DEA for subclass DMUs	70
6.2.1	Subclasses of season W17	71
6.2.2	Subclasses of season W16	72
6.2.3	Subclasses of replenishment products	73
6.3	Summary of results	73
7	Conclusion	77
7.1	Thesis summary	77
7.2	Summary of findings	78
7.3	Recommendations	79
7.4	Further research	80
7.5	Achievement of objectives	80

List of references	83
A Efficiency scores per store format	91
B Efficiency scores per region	105

List of Reserved Symbols

Symbol	Meaning
i	Symbol used to denote the i^{th} input from the set of m inputs
r	Symbol used to denote the r^{th} output from the set of s outputs
j	Symbol used to denote the j^{th} DMU from the set of n DMUs
u_r	Symbol used to denote the weight attributed by DEA to output r
v_i	Symbol used to denote the weight attributed by DEA to input i
x_{ij}	Symbol used to denote the value of input i for DMU j
y_{rj}	Symbol used to denote the value of output r for DMU j
θ	Symbol used to denote the efficiency coefficient or score
θ_{CRS}	Symbol used to denote the efficiency score under constant returns to scale (CRS)
$\bar{\theta}_{CRS}$	Symbol used to denote the average efficiency score under CRS
σ_{CRS}	Symbol used to denote the standard deviation of efficiency scores under CRS
θ_{VRS}	Symbol used to denote the efficiency score under variable returns to scale (VRS)
$\bar{\theta}_{VRS}$	Symbol used to denote the average efficiency score under VRS
σ_{VRS}	Symbol used to denote the standard deviation of efficiency scores under VRS
ϕ	Symbol used to denote the (output-oriented) efficiency score
λ_j	Symbol used to denote the weight attributed by DEA to DMU j
ε	Symbol used to denote a non-Archimedean less than any real positive number
S_i^-	Symbol used to denote the slack variable of input i
S_r^+	Symbol used to denote the slack variable of output r

List of Acronyms

BE	Base exclusive
CRS	Constant returns to scale
DC	Distribution centre
DEA	Data envelopment analysis
DMU	Decision-making unit
ERS	Efficiency reference set
IBT	Inter-branch transfer
LDOW	Last day of the week
LP	Linear programming
NDRS	Non-decreasing returns to scale
NIRS	Non-increasing returns to scale
ROS	Rate of sales
RTS	Returns to scale
SKU	Stock keeping unit
VRS	Variable returns to scale

List of Figures

1.1	A schematic representation of the generic processes in a typical supply chain. . .	1
1.2	A schematic representation of the typical channel of distribution.	2
1.3	A schematic representation of the distribution network for the Retailer	3
1.4	Standard merchandise classification hierarchy in retailers	4
1.5	A schematic representation of the generic input and output process.	5
3.1	Efficient frontier of constant returns to scale	29
3.2	Efficient frontier versus the regression line	30
3.3	Efficient frontier of input-orientation under constant returns to scale	31
3.4	Efficient frontier of output-orientation under constant returns to scale	33
3.5	Efficient frontier of variable returns to scale with all RTS	35
3.6	Efficient frontier of non-increasing returns to scale	35
3.7	Efficient frontier of non-decreasing returns to scale	35
3.8	Efficient frontier of input-orientation under variable returns to scale	36
3.9	Slack variables with regards to the efficient frontier of VRS	37
3.10	Efficient frontier of output-orientation under variable returns to scale	39
3.11	Technical efficiency and scale efficiency under constant and variable returns to scale	40
4.1	Product classification hierarchy	45
4.2	The life cycle of a fashion product expressed through the sales over time. . . .	51
4.3	The life cycle of a replenishment product expressed through the sales over time. .	51
6.1	The number of subclasses within each product class	70

List of Tables

3.1	Data of ten DMUs given input x_1 and output y_1	29
3.2	The efficiency scores and the ERS of ten DMUs under CRS	31
3.3	The efficiency scores and the ERS of input-orientation under VRS	36
3.4	The efficiency scores and the ERS of output-orientation under VRS	39
4.1	An example of the unique data entries of a store.	44
4.2	Attributes of seasons W17, W16 and 00.	44
4.3	The percentage of stores of the Retailer located in 15 regions over southern Africa.	46
4.4	The store format categories of the considered stores of the Retailer.	46
4.5	An example of four week's data entries for store H8793 of season W17.	47
4.6	The number of subclasses and the total number of units inflow per season.	50
4.7	The percentage total regular sales and total promotional sales per season.	51
5.1	The correlation coefficients of the relationship between turnover and rate of sales.	55
6.1	Average efficiency scores of store formats under CRS and VRS	60
6.2	Regression statistics of inputs under CRS.	61
6.3	Regression statistics of inputs under VRS.	61
6.4	Regression statistics of outputs under CRS.	61
6.5	Regression statistics of outputs under VRS.	61
6.6	Regression statistics of outputs under CRS without ROS.	62
6.7	Regression statistics of outputs under VRS without ROS.	62
6.8	Regression statistics for the regression models.	62
6.9	The average inputs and average outputs of each store format	62
6.10	The efficiency scores of the top 20 turnover stores of products from season W17 under CRS and VRS	63
6.11	The efficiency scores of the bottom 20 turnover stores of products from season W17 under CRS and VRS	64
6.12	The efficiency reference set of the bottom 20 turnover stores from season W17	65
6.13	Average efficiency scores of stores in regions of season W17 under CRS and VRS	66

6.14	The efficiency scores of the top 20 turnover stores of products from season W16 under CRS and VRS	67
6.15	The efficiency scores of the bottom 20 turnover stores of products from season W16 under CRS and VRS	67
6.16	Average efficiency scores of stores in regions of season W16 under CRS and VRS	68
6.17	The efficiency scores of the top 20 turnover stores of replenishment products under CRS and VRS	69
6.18	The efficiency scores of the bottom 20 turnover stores of replenishment products under CRS and VRS	69
6.19	Average efficiency scores of stores in regions of replenishment products under CRS and VRS	70
6.20	Efficiency scores of W17 subclasses under CRS and VRS	71
6.21	Efficiency scores of W16 subclasses under CRS and VRS	72
6.22	Efficiency scores of replenishment subclasses under CRS and VRS	73
A.1	Efficiency scores of stores of store format “B” under CRS and VRS	91
A.2	Efficiency scores of stores of store format “C” under CRS and VRS	92
A.3	Efficiency scores of stores of store format “D” under CRS and VRS	93
A.4	Efficiency scores of stores of store format “E” under CRS and VRS	94
A.5	Efficiency scores of stores of store format “E” under CRS and VRS (continued) .	95
A.6	Efficiency scores of stores of store format “F” under CRS and VRS	96
A.7	Efficiency scores of stores of store format “F” under CRS and VRS (continued) .	97
A.8	Efficiency scores of stores of store format “G” under CRS and VRS	98
A.9	Efficiency scores of stores of store format “G” under CRS and VRS (continued) .	99
A.10	Efficiency scores of stores of store format “G” under CRS and VRS (further continued)	100
A.11	Efficiency scores of stores of store format “H” under CRS and VRS	101
A.12	Efficiency scores of stores of store format “H” under CRS and VRS (continued) .	102
A.13	Efficiency scores of stores of store format “H” under CRS and VRS (further continued)	103
B.1	Efficiency scores of stores in the Southern Namibia region of season W17	105
B.2	Efficiency scores of stores in the Northern Namibia region of season W17	106
B.3	Efficiency scores of stores in the Swaziland region of season W17	106
B.4	Efficiency scores of stores in the Botswana region of season W17	107
B.5	Efficiency scores of stores in the Cederberg region of season W17	108
B.6	Efficiency scores of stores in the Kwenya region of season W17	109
B.7	Efficiency scores of stores in the Emfuleni region of season W17	110

B.8	Efficiency scores of stores in the Langeberg region of season W17	111
B.9	Efficiency scores of stores in the North West region of season W17	112
B.10	Efficiency scores of stores in the Free State region of season W17	113
B.11	Efficiency scores of stores in the Lesedi region of season W17	114
B.12	Efficiency scores of stores in the Gauteng region of season W17	115
B.13	Efficiency scores of stores in the Limpopo region of season W17	116
B.14	Efficiency scores of stores in the Thekwini region of season W17	117
B.15	Efficiency scores of stores in the Tugela region of season W17	118
B.16	Efficiency scores of stores in the Southern Namibia region of season W16	119
B.17	Efficiency scores of stores in the Northern Namibia region of season W16	119
B.18	Efficiency scores of stores in the Swaziland region of season W16	120
B.19	Efficiency scores of stores in the Botswana region of season W16	120
B.20	Efficiency scores of stores in the Cederberg region of season W16	121
B.21	Efficiency scores of stores in the Kweni region of season W16	122
B.22	Efficiency scores of stores in the Emfuleni region of season W16	123
B.23	Efficiency scores of stores in the Langeberg region of season W16	124
B.24	Efficiency scores of stores in the North West region of season W16	125
B.25	Efficiency scores of stores in the Free State region of season W16	126
B.26	Efficiency scores of stores in the Lesedi region of season W16	127
B.27	Efficiency scores of stores in the Gauteng region of season W16	128
B.28	Efficiency scores of stores in the Limpopo region of season W16	129
B.29	Efficiency scores of stores in the Thekwini region of season W16	130
B.30	Efficiency scores of stores in the Tugela region of season W16	131
B.31	Efficiency scores of stores in the Southern Namibia region of replenishment products	132
B.32	Efficiency scores of stores in the Northern Namibia region of replenishment products	132
B.33	Efficiency scores of stores in the Swaziland region of replenishment products . . .	133
B.34	Efficiency scores of stores in the Botswana region of replenishment products . . .	133
B.35	Efficiency scores of stores in the Cederberg region of replenishment products . . .	134
B.36	Efficiency scores of stores in the Kweni region of replenishment products	135
B.37	Efficiency scores of stores in the Emfuleni region of replenishment products . . .	136
B.38	Efficiency scores of stores in the Langeberg region of replenishment products . .	137
B.39	Efficiency scores of stores in the North West region of replenishment products . .	138
B.40	Efficiency scores of stores in the Free State region of replenishment products . . .	139
B.41	Efficiency scores of stores in the Lesedi region of replenishment products	140
B.42	Efficiency scores of stores in the Gauteng region of replenishment products . . .	141

B.43 Efficiency scores of stores in the Limpopo region of replenishment products . . .	142
B.44 Efficiency scores of stores in the Thekwini region of replenishment products . . .	143
B.45 Efficiency scores of stores in the Tugela region of replenishment products	144

CHAPTER 1

Introduction

Contents

1.1	The retail supply chain	1
1.2	The distribution network of a retail chain	2
1.3	Stages of planning	3
1.4	Productivity or efficiency	5
1.5	Benchmarking	6
1.6	Data envelopment analysis	6
1.7	Problem description	7
1.8	Scope of thesis	8
1.9	Thesis objectives	8
1.10	Thesis structure	9

Retailing may be defined as “all [the] activities involved in selling goods or services directly to final consumers for their personal use” [55]. This general definition encompasses the commonalities of diverse establishments partaking in these activities, known as retailers. A vast majority of businesses around the world are in the retail industry, with the top 250 retailers identified by Deloitte’s *Global Powers of Retailing 2018* aggregating a retail revenue of US\$ 4.4 trillion in the 2016 financial year [28]. Five South African companies have been ranked in this listing, of which three (Steinhoff International Holdings, the SPAR Group Limited and Woolworths Holdings Limited) were identified as being in the top 50 fastest growing retailers based on the financial years 2011 to 2016 [28].

1.1 The retail supply chain

Large retailers and retail chains¹ trade with thousands of products each and every day. The supply chain process describes how these retailers and vendors ensure that products are available in stores² when customers want it, how retailers respond to consumer needs, introduce new merchandise, and minimise stock-outs while also maintaining cost-efficiency [54].

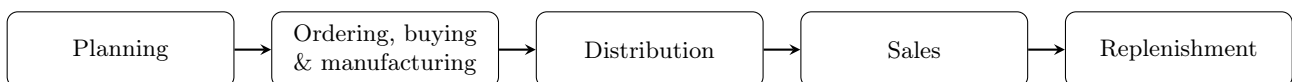


FIGURE 1.1: A schematic representation of the generic processes in a typical supply chain.

¹A retail chain is a retail outlet that has stores in multiple locations.

²Stores may be considered as physical or on-line entities.

The processes of the supply chain for an apparel retail business is summarised in Figure 1.1. This process aligns and integrates the processes of the suppliers, manufacturers, warehouses and distribution centres, transportation entities and stores to make sure that customer needs are met on time, in the right location and the right quantity [54].

Chain stores in the retail industry follow either a push-based product supply chain or a pull-based product supply chain process. The differentiating factor between these two supply chain processes is the way in which planning, manufacturing and inventory management is conducted.

A pull-based supply chain is a demand-driven process, where the manufacturing and distribution processes are directed by actual customer demand through orders from retail outlets. This reduces the inventory carried by firms as supply is order-specific, which requires information of customer demand to flow quickly to distribution and manufacturing [67]. In short, stock is pulled from the customers' side through the supply chain.

A push-based supply chain is driven by demand forecasts as determined centrally by the retailing entity. The emphasis is on the entity to decide when and how much stock is sent to the stores. The demand forecasts are based on present inventory positions and historical performance. In this case, stock is pushed down the supply chain to satisfy expected demand. Therefore, the pace of manufacturing, distribution decisions and priorities are set centrally by the business rather than by the stores [67].

A push-based supply chain process is not as dynamic as the pull-based process because it is not always based on the most current customer demand. It takes longer for an entity to react to change when using a push-based system [67]. However, less safety stock is needed in the system as no extra stock is needed in the system to account for unknown and unexpected orders from the stores to replenish stock.

1.2 The distribution network of a retail chain

Retailers provide a link for products to be transferred from its initial state from suppliers or manufacturers to where the final product is received by customers [14]. Figure 1.2 shows how products flow through four key role players, which are the manufacturer, the wholesaler, the retailer and finally the consumer [8, 31, 54].

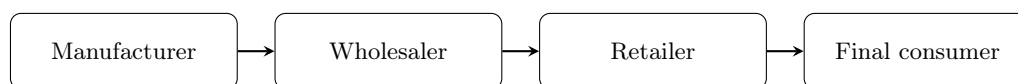


FIGURE 1.2: A schematic representation of the typical channel of distribution.

Manufacturers and wholesalers produce the goods and supply the retailer with products that are then sold to customers for personal, family and household use [8]. Planning and ordering the right goods is a process which is done well in advance before products reach shelves in stores.

The activities associated with the supply chain process requires extensive coordination and planning of resources to ensure the finished product is delivered at the right place and time to clients. These activities include sourcing of parts and raw materials, manufacturing and assembly, inventory control and warehousing, management of orders, distribution, delivery of products to customers, and the monitoring of goods throughout the supply chain process [56].

The retailer considered in this study, which will subsequently be referred to as the Retailer, is a prominent clothing retailer (as well as other products) and makes use of the distribution network in Figure 1.3. This distribution network relies on two processes that allows the Retailer

to be the link between suppliers and consumers. These processes are known as the planning phase and the allocation phase. During the planning phase, decisions are made as to what products to order, the order quantity and the frequency of orders. Decisions on forecasts for sales and thoughts on products for the next season are established and orders are implemented based on these plans, until it arrives at a distribution centre. Once it arrives at the distribution centre (DC) months after the order is placed, the Retailer then starts the allocation phase of distribution. The allocation phase is the period where the products are stored, sorted, packed and distributed to various outlets and sold to customers.

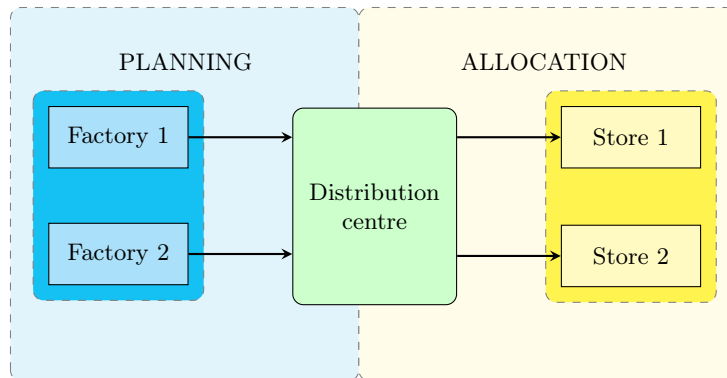


FIGURE 1.3: A schematic representation of the distribution network for the Retailer.

The Retailer follows a push-based supply chain, so information about the performance of products and stock are updated as the sales season progresses and until new sales data become available to forecast and plan for the following season. The Retailer places orders at factories from about 6 to 10 months before the start of the sales season, and once the products arrive at the DCs, the Retailer will start the allocation process. Once allocation decisions are taken, it takes about 2 weeks to deliver that stock to all stores.

1.3 Stages of planning

There are multiple planning stages throughout the supply chain that retailers undertake to ensure that the customer's needs are met. Company directors will ask themselves how they will achieve their goals of satisfying customers by considering the decisions they will make concerning these stages. These stages include merchandise planning, assortment planning, allocation planning and replenishment planning.

Merchandise planning is a process with the objective of satisfying customer needs while achieving a retailer's financial goals [54, 71, 92]. The primary goal of any retailer is to sell merchandise. The retailer does this by offering the right product in the right place and time, in the correct quantity and at the right price so as to meet the company's financial goals [54]. The retailer identifies the categories and markets of products it wants to stock (which is often based on the needs of consumers), where the items will be sourced from, or how they will be produced.

Figure 1.4 shows a standard merchandise hierarchy or classification scheme that categorises the way in which some retailers organise and differentiate the nature of their products [5, 20, 54]. This classification groups products into categories of similar attributes. The hierarchy specifies the market (i.e. the department), the collection (i.e. the season), the style and family (such as casual wear and T-shirts), the article (which uniquely identifies that product style), the

colour and the size of products [20]. This ultimately refines products to a stock-keeping unit (SKU) level, but it will be very difficult to determine what units to procure without the initial grouping of items into categories [5, 54]. A retail store can have this structure of products stocked in its store, i.e. having multiple of each level of the merchandise hierarchy to appeal to all customers [5].

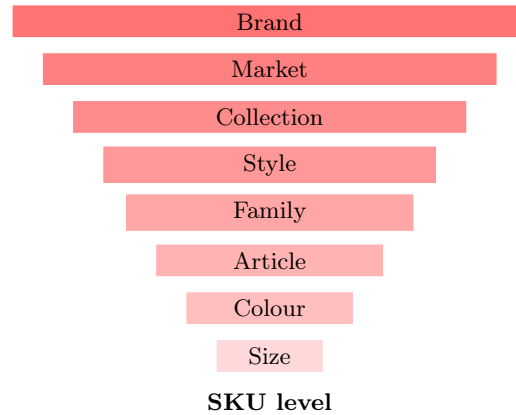


FIGURE 1.4: *Standard merchandise classification hierarchy in retailers.*

Another important stage in the planning process is assortment planning, which establishes the width and depth of each product for the retailer [45]. Clothing retailers will make decisions on the quantity and the assortment of products to stock in their stores. The width of the products is the collection and family of products that a retailer decides to stock, which concerns the higher levels of the merchandise classification hierarchy. The depth of those categories are the styles, colours and sizes of products, which concerns the lower levels of the merchandise hierarchy. The depth of assortment allows customers the opportunity to have variety in a particular product, and the width offers customers variety in the types of products [45].

The following stage in the planning process is the allocation phase. Once the merchandise and assortment planning processes have been completed, the retailer will order, manufacture or buy the products. Once it has been received, the products are allocated to specific locations for sale. This process includes determining the quantity of products being sent to each location, and the mix of products that are allocated for each location. The products are then sorted, packaged and transported from the DCs to stores.

The final stage in the planning process is replenishment. Retail stores make a distinction between replenishment products and fashion products. Replenishment products are in continuous demand throughout the year. These products have relatively stable sales over extended time periods and the demand is predictable. Therefore, an error in forecasting can easily be overcome and replenishment products require continuous monitoring to ensure the inventory levels do not deviate to dangerous levels [92]. Examples of replenishment products are white school shirts, undergarments and socks. Fashion products are products that are only in demand for a relatively short period of time. These products typically have a seasonal life span. It is more difficult to forecast the performance of these products, as it is less flexible to correct forecasting errors. Fashion products have a high demand volatility and is typically not replenished [54]. Examples of fashion products are winter jackets and swimwear.

The replenishment planning stage is only implemented for replenishment items. This stage focuses on the inventory levels of stock throughout the selling season. Inventory data are collected and analysed in order to replenish stock, if necessary, and to aid assortment planning for the following year. The sequence that these planning stages take place differs for different retailers.

1.4 Productivity or efficiency

A system is defined as a set of analogous activities or actions that are performed to create a product, service or result [53, 69]. The components that characterise a system are the inputs and outputs that contribute to the result or outcome, and the process that utilises and produces those inputs and outputs [69]. A generic input and output process of a system is provided in Figure 1.5.

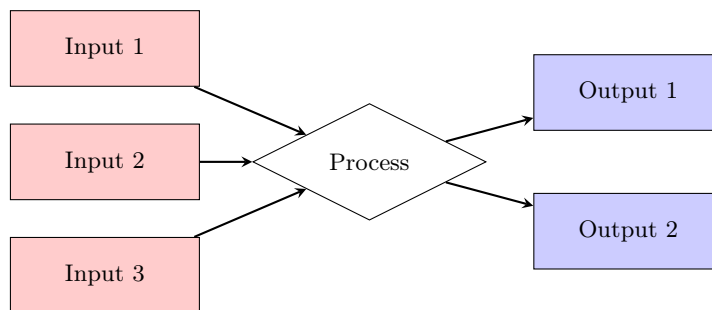


FIGURE 1.5: A schematic representation of the generic input and output process.

Inputs are resources, commodities, information or people used by a system to obtain a desired output [13]. Expenses are often attributed as being inputs, as it is necessary to incur some costs for a system to operate. An output is a tangible or intangible result produced by a process from utilising inputs [53]. This may include the achievement of financial or operational goals for a company. A process is defined as a task, project or a business unit that utilises inputs in some way to achieve an output(s). It is the underlying goal of every company to maximise the output of a system by minimising the level of inputs utilised. Functioning in this way and striving to improve performance is known as productivity, or efficiency³ [53].

Inputs can be utilised at any stage of the distribution network: some inputs may occur in the planning phase, which ultimately affects the processes during the allocation phase, while outputs are determined after the inputs have been employed. Productivity in companies is often measured primarily by the performance of outputs. Financial indicators, profits and returns on investments have traditionally been an indication of how well a retailer is performing, but isolating the performance of outputs does not provide an indication of the productivity of a process's use of inputs and outputs.

The development of technology has enabled companies to make advances in inventory control decisions, merchandise and assortment planning, retail production, distribution and forecasting techniques [71, 84]. These technological advances creates the opportunity for retailing entities to improve performance by producing a greater level of output with a given level of input, or by minimising the level of input needed to produce a given level of output.

Efficiency is the relationship of outputs relative to inputs [53, 82]. Benchmarking is a particularly powerful tool to measure efficiency, since it delineates the potential of a process or system, which will henceforth be referred to as a decision-making unit⁴, to perform at its best relative to best-practice units [70].

³The terminology "efficiency" will be used interchangeably with "productivity" in this study.

⁴The terminology "decision-making units" will be used interchangeably with "service units" in this study.

1.5 Benchmarking

Grinyer and Goldsmith [43] said it best when they defined benchmarking as the continuous process of measuring, comparing and improving processes against the best that can be identified. A benchmark is a standard that others want to imitate, so benchmarking is a tool used to compare and, most importantly, to improve the performance of processes, practices, goods and services [4]. There is a distinction between internal and external benchmarking. Internal benchmarking is the comparison of different processes within a firm, whereas external benchmarking is the comparison between firms within an industry [70].

Benchmarking and other efficiency tools are applied to help the management of companies make informed and insightful decisions, and to optimise or improve the processes within the supply chain [4, 49]. The decisions around supply chain processes and their respective activities can be complex, especially when there are an abundance of data to analyse and interpret.

There are multiple decision support techniques and tools that companies utilise to make sense of their data in a rational manner. These tools and techniques include optimisation techniques and heuristics, simulation models, data mining and warehousing, statistical analyses, and artificial intelligence systems [49]. These tools, including benchmarking, are not once-off analyses that improve a company's performance: it is rather an ongoing process that must be reviewed and repeated to ensure that best practices are maintained and long-term improvement is guaranteed [53].

1.6 Data envelopment analysis

Data envelopment analysis (DEA) is a benchmarking tool first developed by Charnes, Cooper and Rhodes [17] in 1978, who extended upon the work of productive efficiency by Farrell [37] in 1957. This non-parametric linear programming model was developed to evaluate the relative efficiency of the activities of non-profit entities participating in public programs. The aim of the DEA model was to provide a scalar measure of efficiency for each participating unit. DEA models have since been developed and applied to measure the efficiency of other service units. The decision-making units (DMUs) evaluated by the DEA model perform the same function with the same objective by using certain inputs to produce outputs [12]. Efficiency in the context of DEA may be defined as a ratio of output to input, where more output per unit of input implies greater efficiency [82].

An optimum or absolute state of efficiency is achieved when the greatest possible output per unit of input is reached, and it is not possible to become more efficient with current technology or without making changes to the production process. However, optimum efficiency cannot be determined for service units, as information of maximum output is unknown and limited when considering efficiency over multiple outputs. It may also be due to the current technology available and the production process used, the scale or size of the service unit or how well the production process is managed. The DEA model can identify the output-to-input ratio of many DMUs relative to other DMUs and determine that one unit is more or less efficient than another unit. This identifies DEA models as benchmarking tools for relative efficiency [82].

DEA works with inputs, i , and outputs, r , where $i \in \{1, 2, 3, \dots, m\}$ and $r \in \{1, 2, 3, \dots, s\}$, to evaluate how well outputs of a DMU performs given a certain set of inputs. The efficiency score⁵ of DMU j , denoted by θ_j , of a single input ($i = 1$) and a single output ($r = 1$) can be

⁵The symbol θ is used to denote the efficient score and is consistent with the original DEA literature.

expressed as output 1 divided by input 1. In general, the efficiency of DMU j with multiple inputs ($i \geq 1$) and/or multiple outputs ($r \geq 1$) is expressed as the weighted sum of the outputs over the weighted sum of the inputs,

$$\theta_j = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}}, \quad i \in \{1, \dots, m\} \text{ and } r \in \{1, \dots, s\}, \quad (1.1)$$

where y_{rj} is the value of output r on DMU j , x_{ij} is the value of input i on DMU j , u_r is the weight assigned to output r and v_i is the weight assigned to input i . These weights are objectively determined from the observed data of the inputs and outputs.

Best-practice DMUs achieve a relative efficiency score of $\theta = 1$ or 100%, which means that no other unit is operating more efficiently than this unit given their combination of inputs and outputs. It does not, however, mean that there is no opportunity for the efficient unit to perform any better. DMUs with an efficiency score that is less than 1 ($0 \leq \theta < 1$) are identified as inefficient units. These units are strictly inefficient compared to other DMUs. DEA seeks the maximum efficiency score, which tends to understate rather than overstate a DMU's inefficiency. This means that an inefficient DMU may be less efficient than is identified by the DEA model [82].

An efficiency score of θ may be interpreted as the level of input consumption that should be achieved in order to become efficient. This means that one way for a DMU to become efficient is to reduce its inputs to $(\theta \times 100)\%$ of its current level. Another way to interpret this is that a DMU is using $((1 - \theta) \times 100)\%$ excess resources as determined by the DEA model when compared to efficient units.

1.7 Problem description

Efficiency is an ideal that many organisations strive towards, and it is an ideal that is very relevant. This is evident from the European Union's (EU) decision to identify resource efficiency as a flagship initiative for its 2020 strategy towards a "green economy" [11]. The opportunity to reduce the cost of time, money and inputs is something that appeals to all industries. It is almost always possible to run processes more effectively, which can be done by minimising waste and ensuring that the best result is produced all the time.

This thesis aims to analyse the efficiency of decision-making units in the retail environment by investigating how inputs and outputs are utilised for different processes of a major retailer, known as the Retailer. This analysis will focus on the benchmarking of DMUs using DEA as the efficiency measure. The Retailer provides ample real data to use. The number of considered service units is often limited in other studies. The provided sample data is of such a size that allows for testing of DEA on a large scale compared to other studies, which is what sets this study apart from other studies.

Extensive research has been performed on the efficiency of resource allocation in literature [49], but it is important to look at the efficiency of entire systems in the distribution network [70], how the performance of each system is compared to similar systems within a retailer's supply chain, and how productivity can be improved for each system based on its use of inputs and outputs.

This can be done by identifying inputs and outputs from various stages in the distribution network instead of isolating the scope to efficient resource allocation.

1.8 Scope of thesis

The scope of this thesis will focus on data from the baby boys' outerwear department of a prominent South African retailer. The inputs and outputs for this study will be from different stages of the distribution network, and will investigate the efficiency of fashion and replenishment products. Various levels of the merchandise hierarchy will be DMUs for the DEA models, and the results will be grouped together for comparability.

This thesis will also investigate the distinct orientations of DEA, and results of DEA for different returns to scale will be determined and compared. This thesis will comment on the results of DEA when a large and comprehensive dataset is considered, and what the corresponding effect this added discriminatory power has on the accuracy of the results.

This thesis has relevance to literature as there are not many studies done on calculation group⁶ sizes as large as the dataset obtained from the Retailer, so there is greater discriminatory power as a result [30, 66]. It is also beneficial for the management of the Retailer, as it allows the Retailer to have control of its processes and to make changes identified by the DEA models to perform better. This study will affect the way planning is done at the start of the supply chain, and ultimately lead to informed decision-making for processes throughout the distribution network of the Retailer.

1.9 Thesis objectives

The problem statement in this thesis will be investigated by addressing the following objectives:

Objective I: Understanding DEA as an efficiency measure

- a Explain the importance of efficiency.
- b Investigate how to measure efficiency using DEA.

Objective II: Collecting relevant data for DEA from the Retailer

- a Collect and analyse relevant data to determine inputs and outputs.
- b Validate, clean and describe the collected data.

Objective III: Specification of variables and calculation groups for DEA

- a Identify and describe inputs and outputs.
- b Identify and describe the different service units.
- c Describe the relevant grouping criteria.

⁶A calculation group is the set of observations against which a DMU's efficiency is calculated, i.e. it is the set of DMU benchmarks used in the DEA model [29].

Objective IV: Analysis of DEA results

- a Explain and validate the results of the DEA for all grouping criteria.
- b Identify decision-making units as efficient and inefficient based on the DEA efficiency scores.
- c Identify trends in the efficiency of particular groups of service units.

1.10 Thesis structure

This chapter started with an overview of the retail industry and the supply chain of retailers. Efficiency and benchmarking were also discussed. A problem description and the relevance of investigation into this problem were provided, followed by the scope and objectives for this thesis. Chapter 2 details the relevant literature on efficiency and data envelopment analysis. Chapter 3 contains the methodology and underlying principles of DEA used to build the models for the Retailer.

Chapter 4 provides a comprehensive validation and analysis of data and Chapter 5 describes the grouping of the data for comparability and to identify informative results and trends from service units. Chapter 6 contains a summary of the results obtained from the DEA models. This thesis is concluded with Chapter 7, which provides final remarks on the objectives achieved, the results from this study and ideas for further research.

CHAPTER 2

Literature review

Contents

2.1	Productivity and benchmarking measures	12
2.1.1	<i>Regression analysis</i>	12
2.1.2	<i>Stochastic frontier analysis</i>	15
2.1.3	<i>Goal programming</i>	15
2.2	Extensions of DEA	16
2.2.1	<i>The CCR ratio model</i>	16
2.2.2	<i>The BCC model</i>	16
2.2.3	<i>The multiplicative DEA models</i>	17
2.2.4	<i>The additive DEA models</i>	17
2.2.5	<i>Slack-based measure of efficiency</i>	17
2.2.6	<i>Super-efficiency and cross-efficiency</i>	18
2.3	Applications of DEA	18
2.4	Efficiency measurement in the retail industry	19
2.5	Input and output mix	21
2.6	The benefits of DEA	22
2.7	The shortcomings of DEA	22

Productivity may be improved by managing and monitoring multiple company components. There are two universal components related to productivity that are relevant to this study, which are often confused due to the boundaries of their meaning. Effectiveness is the ability of an entity to set and achieve its goals and objectives. Efficiency, which may be used interchangeably with productivity, is the ability to produce the outputs or services with the minimum required resource level. Alternatively, effectiveness can be seen as doing the “right job” and efficiency as doing the “job right” [82].

This chapter begins with an investigation of various benchmarking tools and productivity measures in § 2.1, and follows with a description of the literature that is relevant to this thesis in § 2.2. The brief description of the applications of DEA in different industries is provided in § 2.3, which provides insight into the versatility of DEA as a benchmarking tool. Studies on the application of DEA as it pertains specifically to the retail industry is given in § 2.4. A description of the importance of variable selection of inputs and outputs, and the accompanying discriminatory power, is given in § 2.5. The chapter then concludes with the benefits and shortcomings of using DEA in § 2.6 and § 2.7.

2.1 Productivity and benchmarking measures

Productivity is naturally defined as the ratio of outputs to inputs, where larger values indicate better performance. Measuring productivity concerns the performance of all production factors, and not just performance in terms of land or labour, which is known as partial productivity. There have been many advances and techniques in how productivity between firms is measured. These techniques differ by the information that they require and the assumptions made to produce productivity measurements [21].

Productivity measures have primarily been used under assumptions that are rarely applicable in reality, such as the homogeneity of the nature of inputs and outputs [21]. Other methods, like index number methods and least squares econometric methods, perform under the assumption that all firms are efficient. Thus, it is important to relax these assumptions to determine productivity based on raw data.

2.1.1 Regression analysis

Regression analysis is the study of the mathematical relationship between a dependent variable and one or more independent variables. Simple linear regression is used when there is a linear relationship between the dependent variable and one independent variable, whereas multiple linear regression (also simply known as “multiple regression”) is the relationship with more than one independent variable. Regression relies on this mathematical relationship to predict the average or mean or expected value of the dependent variable when the values of the independent variables are known [91, 94]. Regression analysis is often applied to the retail industry, predominantly in the forecasting of sales [72]. Regression is considered to be one of the most frequently used techniques for forecasting, despite the existence of more modern forecasting methods [1, 57].

Multiple regression

Let Y be the value of the dependent variable and X_i be the value of the i^{th} independent variable. The linear model relating Y to the set of independent variables is of the form

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k + \varepsilon, \quad (2.1)$$

where β_0 is the intercept, β_i are the unknown parameters associated with X_i for all i , and ε is an error term that represents the fact that the actual value of Y may not be equal to $\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k$. The error term has a mean of 0 and should follow a normal distribution. The parameter β_i may be seen as the increase in Y if the value of the i^{th} independent variable is increased by 1 and all other independent variables remain constant.

The values of β_i are unknown and are usually estimated from sample data as $\hat{\beta}_i$. Let \hat{Y} be the predicted value of the dependent variable. The value of \hat{Y} can be estimated by the regression line

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \dots + \hat{\beta}_k X_k, \quad (2.2)$$

Equation (2.2) is known as the least squares regression equation.

The estimates for $\hat{\beta}_i$ may be estimated by minimising the sum of the squared errors (SSE) of all the observations. In other words, let $\mathcal{J} = \{1, 2, \dots, j, \dots, J\}$ be a set of observations, and

minimise

$$\sum_{j \in \mathcal{J}} \varepsilon_j = \sum_{j \in \mathcal{J}} (Y_j - \hat{Y}_j)^2 \quad (2.3)$$

$$= \sum_{j \in \mathcal{J}} (Y_j - \hat{\beta}_0 - \hat{\beta}_1 X_{1j} - \hat{\beta}_2 X_{2j} - \dots - \hat{\beta}_k X_{kj})^2, \quad (2.4)$$

where ε_j is the error of the j^{th} observation, Y_j is the dependent variable of the j^{th} observation, \hat{Y}_j is the j^{th} predicted value and X_{ij} is the value of the i^{th} independent variable for the j^{th} observation.

The accuracy of the regression model may be determined using a number of measures and statistics. One such measure is the coefficient of determination, or R^2 , which measures how well the regression line fits the data. The value of R^2 may also be seen as the percentage of variation in Y (the dependent variable) explained by the independent variable(s), and hence $1 - R^2$ is the percentage of variation in Y not explained by the independent variable(s). An R^2 value that is close to 1 indicates a good fit of the regression line. An increase in the number of independent variables to the regression equations may lead to an increase in the value of R^2 [94].

The inclusion of independent variables in multiple regression should be tested for suitability. This is validated by testing the hypothesis

$$H_0 : \beta_i = 0, \text{ against} \quad (2.5)$$

$$H_a : \beta_i \neq 0. \quad (2.6)$$

The null hypothesis, H_0 , and the alternative hypothesis, H_a is tested for each independent variable [94]. If β_i is 0, it means that the i^{th} independent variable does not have a significant effect on the dependent variable Y with the other independent variables included in the regression equation. If the null hypothesis is rejected, it implies that the independent variable does have a significant effect on the independent variable.

These hypotheses are tested by computing

$$t = \frac{\hat{\beta}_i}{\text{StdErr}(\hat{\beta}_i)}, \quad (2.7)$$

where $\text{StdErr}(\hat{\beta}_i)$ is the standard error of β_i , which measures the amount of uncertainty present in the estimation of β_i . The null hypothesis is rejected at a significance level of α , if $t \geq t_{(\frac{\alpha}{2}, n-k-1)}$, where n is the number of observations and k is the number of independent variables.

In addition to the t -statistic, a p value is also calculated. The p value is given as

$$\text{Probability}(|t_{n-k-1}| \geq |\text{Observed } t\text{-statistic}|), \quad (2.8)$$

where n is the number of observations and k is the number of independent variables. An independent variables is significant when H_0 is rejected for a p value less than a given α value [94].

Multiple linear regression relies upon certain assumptions about the variables in the analysis. Results may not be trustworthy if these assumptions are not met and may lead to serious biases in the interpretation of the results. Regression analysis assumes that the error terms are normally distributed with a mean value of zero [93]. This is important for making inferences about the regression parameters, specifically for significance testing. Another assumption is that standard multiple regression can only make an accurate estimation if the relationship between the dependent variable and the independent variables are linear in nature [64].

Additional assumptions that must be considered are regarding homoscedasticity and heteroscedasticity, autocorrelation and multicollinearity. Homoscedasticity means that the variance of errors is the same or constant over different values of the independent variables, and variance of errors that differs at different values of independent variables is known as heteroscedasticity [64]. Multiple regression assumes that the errors terms are homoscedastic. Autocorrelation means that the error term of one observation has an influence on an error term corresponding to another observation [9]. Multiple regression assumes that no autocorrelation is present. The assumption also holds that no multicollinearity is present, which implies that there is no correlation between two or more different independent variables [93].

Regression analysis versus DEA

Regression analysis is among the most widely used comparative efficiency techniques, apart from DEA itself [26]. The prevailing impression in literature is that regression analysis and DEA are alternative and equivalent algorithms for relative efficiency, although the two techniques produce different efficiency results.

The advantages of the regression analysis approach is that there are a number of statistical tests to investigate the validity of the model, and regression is able to assign a negligible weight to variables that are not relevant. Regression analysis identifies a weight that is consistent for all observations, whereas DEA identifies weights that may differ for different observations [26]. Linear regression, in the context of production theory, produces an unbiased estimate of the parameters of a cost function, and requires at least one observation that performs efficiently to produce a frontier. Regression analysis makes use of a least-squares algorithm to fit an average line as a frontier, whereas DEA uses linear programming to fit an efficient frontier.

Regression analysis, unlike DEA, makes assumptions on the stochastic properties of the data, such as the distributions of the observed data points. This advantage allows for empirical and statistical significance testing on competing variables. However, if the variables of the observed data are highly collinear, regression analysis may confront the problem of multicollinearity, which will make modelling difficult.

Cubbin and Tzanidakis [26] compared regression analysis with DEA, and concluded that regression analysis is beneficial when comparing different companies, and for large samples, DEA is good at identifying poor performance. Both tools are potentially useful for comparative efficiency analysis. To avoid inference and biased results for either of the techniques, samples that contain enough observations to define a frontier adequately are recommended when investigating relative efficiency. Cubbin and Tzanidakis [26] favoured regression analysis above DEA for the fact that statistical testing is possible and that there are greater opportunities for bias in DEA than for regression analysis.

The reasons for pursuing DEA is that it allows for environmental or non-controllable variables to be included in the model, and does not make assumptions about the stochastic properties of the observed data. Additional benefits of DEA over regression analysis is that DEA can readily handle multiple inputs and multiple outputs, where regression analysis can readily handle either multiple inputs or multiple outputs. DEA also does not require the specification of a functional form to be fitted [26]. DEA provides direction for how to improve efficiency, which regression does not provide [75].

2.1.2 Stochastic frontier analysis

Stochastic frontier analysis (SFA), unlike DEA, is a parametric approach that hypothesises a production function and uses the available data to estimate the parameters of that function by using the entire set of DMUs [21]. DEA and SFA are both frontier methods that determine relative efficiency of DMUs.

SFA is able to separate random noise from the efficiency, where DEA incorporates the noise as part of the efficiency score. SFA is also popular for cases of panel or time-varying data and allows for the construction of confidence intervals and the formal statistical testing of hypotheses [46]. The formulation of this model makes use of maximum-likelihood methods and makes assumptions on the distribution of the data. Although there are different underlying assumptions for both models, a study by Cordeiro *et al.* [25] found that the technical efficiency scores of SFA strongly correlated with the results of DEA. The study considered 299 DMUs in 19 different industries [25].

DEA suffers from statistical limitations by not providing fit statistics that can be used for statistical inferences, such as p -value statistics. Although SFA explicitly takes these stochastic properties of the data into account, the advantage of DEA is that the model returns unit-specific data and information of returns to scale and changes in productivity, whereas SFA reveals overall sample-based information. Since there is confidence in the correlation of the two approaches [25], the econometric technique known as SFA will not be considered.

2.1.3 Goal programming

Goal programming is a powerful multi-objective tool that, like regression, may be used as a supplementary technique to enhance the capabilities of DEA [79]. The benefit of DEA models is that a benchmark is established for inefficient DMUs. These benchmarks leave management with identifying the inefficiencies, and goal programming is an additional tool that enables decision makers to create plans for the future with the results from DEA.

Stewart [83] proposed a goal program-based benchmarking which incorporates the efficiency scores from a DEA model with a multi-objective problem to project inefficient (and efficient) DMUs onto a most preferred point on the efficient frontier, based on the goals set by a decision maker [79, 83]. The approach was first proposed by Golany [41], who viewed the technique as an interactive multi-objective linear program (MOLP) that would generate a set of efficient points for a DMU to consider. This started the discussion of the integration of DEA with MOLP methods [95].

The idea of this approach is that the decision maker sets the aspiration levels for the inputs and outputs of a DMU (assuming that a DMU has control over its inputs and outputs), and using goal programming to find benchmarks for the considered DMU (these benchmarks being on the efficient frontier) that will satisfy the goals of the decision maker as closely as possible. The benchmarks are a linear combination of existing DMUs, and because the achievement of the goals must be satisfied as closely as possible, this will ensure that the solution to the problem is on the efficient frontier [83].

The purpose of this link between DEA and goal programming is to ensure that decision-making units are controlled by the decision makers, and that DEA can identify inefficiencies, and goal programming can satisfy the goals of management. This provides decision makers the opportunity to bridge the state of monitoring and control to planning for the future with multi-criteria decision analysis [83].

2.2 Extensions of DEA

The purpose of DEA models as a benchmarking tool is to identify best-practice service units from a set of service units, all of which form a “best-practice frontier” [22]. A service unit that does not perform on the best-practice or efficient frontier will be situated below the frontier region, known as an envelope [7]. The diverse DEA models vary by the forms of the efficient frontiers, and other factors such as the perspective of orientations and the returns to scale [78].

Various extensions of the standard DEA model have been developed comprehensively in literature. The models have different orientations, despite the fact that all the models address managerial issues and provide information that is useful to its constituents [78]. The most common DEA models are the CCR ratio model [17] and the BCC model [6]. The multiplicative models, C^2S^2a [18] and C^2S^2b [19], as well as the additive model, C^2GS^2a [16] and the Cone Ratio model [15], are also a few popular extensions of the original DEA model.

2.2.1 The CCR ratio model

The CCR ratio model, named after the developers Charnes, Cooper and Rhodes [17], was concerned with evaluating public programs and developing measures for decision making efficiency. Decision-making units (DMUs) were identified as non-profit programs with common outputs and inputs [17]. There was an understanding in the study that due to technological constraints¹, it may not be possible to calculate true efficiency, so a scalar measure for ‘relative efficiency’ was developed. The CCR measure of efficiency for a DMU was proposed as the maximum of a ratio of weighted outputs to weighted inputs, subject to the condition that similar ratios of every considered DMU be less than or equal to unity [17].

This approach became popular for its non-parametric nature of the data and by how a scalar measure can be determined from this model. Additional benefits of this model is that the most favourable weighting allowed by the constraints and observed data will maximise the ratio between outputs and inputs. This means that there is no other set of common weights that will give a more favourable relative efficiency score with the given observations and constraints, and there is no requirement of *a priori* specification of weights. The study of the CCR ratio model also detailed solving the dual of the linear program, which is described further in Chapter 3, and also provided the notion that the conditions for efficiency in DEA are also the conditions for Pareto efficiency. The CCR model is the simplest form of DEA, and therefore forms the basis of all other DEA models that have been developed since.

2.2.2 The BCC model

The BCC model was named after the developers Banker, Charnes and Cooper in 1984 [6]. The BCC model is based on the same underlying assumptions and conditions as the CCR model. The development of the BCC model contributed towards a distinction made between technical and scale efficiencies, and hence a new and separate change to the formulation of the CCR model was introduced to include efficiencies at different returns to scale.

The formulation of the CCR ratio model only takes constant returns to scale into consideration, which only measures technical efficiency. The BCC model took increasing and decreasing returns into account in the formulation, and hence scale efficiency is also considered. This means that

¹Technological constraints refer to the limitations of productivity, since advancements in technology are constantly being made.

DMUs can have different productivities and still be considered efficient at different scales. The benefit of this model is that DMUs that were previously technically inefficient are able to be scale efficient. This means that the condition of proportionality of efficiency in the CCR ratio model is relaxed. The formulation of the BCC model is provided in § 3.4.

2.2.3 The multiplicative DEA models

The C^2S^2a and C^2S^2b models, developed by Charnes, Cooper, Seiford and Stutz [18, 19], are two multiplicative DEA models. These multiplicative models are similar to the CCR ratio model except that efficiency is the ratio of weighted product of outputs to the weighted product of inputs. This differs from the CCR ratio model which determines the ratio of the weighted sum of outputs to the weighted sum of inputs.

The C^2S^2b model was developed to be invariant under changes of units in the inputs and outputs, similar to the CCR model and as opposed the C^2S^2a multiplicative model. Efficient DMUs would still be characterised in the same way as in the CCR model. A piecewise log-linear frontier production function is obtained rather than piecewise linear with the CCR model. These models were provided as alternatives to formulating DEA, but further research into the advantages and disadvantages of multiplicative models are scarce.

2.2.4 The additive DEA models

The Cone Ratio model and the C^2GS^2a model [15, 16] are also based on the formulation of the CCR model. The Cone Ratio method provides a substantially generalised version of the CCR model, and can be used for multi-attribute optimisation, cone-ratio and polar cone analysis.

The C^2GS^2a model is concerned with the construction and analysis of the Pareto-efficient frontier production functions, allowing for the possibility of non-linear efficient frontiers. The formulations of both of these models are all based on the CCR ratio model, and may be used in general scenarios of DMUs in different faculties.

2.2.5 Slack-based measure of efficiency

The slack-based measure of efficiency (SBM) model is an efficiency measure proposed by Tone [89] that does not assume proportional changes in inputs or outputs, and has a close connection to the models that will be discussed in § 3.3 and § 3.4. SBM handles input and output slacks directly and not in the radial² sense [24]. In other words, SBM models are able to gauge the depths of inefficiency.

SBM models discard the assumptions of proportional changes in inputs and outputs that other models may assume, since real inputs and outputs do not behave proportionally in reality. Another shortcoming of radial models is that the reporting of efficiency scores is absent of slack variables. This poses a problem if the slacks form an important role of evaluating managerial efficiency. This can result in biased inferences based on misleading efficiency scores [24]. This study considers proportional, as well as variable changes in inputs and outputs.

²Radial efficiency means a proportional changes in inputs and/or outputs.

2.2.6 Super-efficiency and cross-efficiency

DEA identifies efficient DMUs with an efficiency score of one, and it is likely for DEA to identify more than one DMU as efficient, which leads to a tie of efficiency. This is not the case in reality, since DMUs operate at different capacities. Since DEA is not a ranking technique, and in order to break the ties of efficient DMUs, Andersen and Petersen [2] proposed a super-efficiency method. The super-efficiency method allows for the ranking of efficient DMUs, where the efficiency score is not constrained to being strictly equal to one.

DEA models assign unit-specific weights to DMUs, which means that this self-evaluation of DMUs cannot be used to rank the efficiency scores [24]. Cross-efficiency evaluation is the concept of determining cross-efficiency scores of each DMU with the weights of all the DMUs and not only its own weights. These tools are widely used in DEA, specifically in the ranking of efficient DMUs and for comparing the performance of two groups. The disadvantages of using these two methods are that infeasibility can occur when the convexity of DEA models are considered, or the possible existence of alternative optima for weights, which can lead to inconsistent cross-efficiency scores [23, 24].

Super-efficiency is the result of a DEA model where the DMU under consideration is not included in the calculation group of the models. When the considered DMU is not included in the formulation for DEA, the efficient frontier and all the results are calculated without that DMU. It is known in some studies in literature as ‘leverage’, which may be defined as the impact that removing a DMU from the dataset has on the efficiency scores of all other DMUs in the dataset [27]. This is done by calculating the efficiency scores of each DMU, followed by removing one DMU at a time and calculating the efficiency scores of each DMU. This approach is computationally intensive and requires significant computer resources. If there are n DMUs in a sample, then DEA calculations must be performed $n \times (n - 1)$ times to determine the leverage of each DMU in the sample. While leverage may be a good technique to determine the effect of outliers on the efficiency scores, this approach is unfeasible for large datasets, and other measures should be considered if the leverage is required [27].

2.3 Applications of DEA

The nature of these models has led to the widespread application of DEA in many studies spanning multiple faculties. Many organisations and operations have utilised this model using data from their fields of operations. Some common examples of such organisations are financial institutions, such as banks [81] and tax collectors, hospitals, pharmacies and other health care providers [62], academic areas, such as schools, colleges, academic departments and universities [3, 88], housing services, computer system design and software development, manufacturing and regional planning, the operations special forces and other military operations, and even the efficiency of urban agglomerations in the Chinese economy [35]. These are only a few examples of the many fields where data have been utilised to determine efficiency with the aid of DEA [78].

Sherman and Gold [81] studied the operating efficiency of bank branches. Many studies have been done on the efficiency of banks, with traditional performance measures such as return on assets and return on investments being limiting. This study considered four outputs and three inputs. The DMUs for this study were 14 bank branches, and the results from the study were analysed with management to corroborate the observed results. The conclusions made by this study were that DEA was able to accurately locate the inefficient branches, but from the limitations of DEA was not able to locate *all* the inefficient branches. However, the fact

that DEA could direct management to DMUs with real inefficiencies was compensation for that limitation. The other conclusion was that the cause or remedy for inefficient DMUs is not indicated, and that emphasis should be placed on the selection of appropriate inputs and outputs. This thesis will consider more DMUs at a time and compare the results of smaller samples to larger samples.

DEA has been applied to South African public hospitals in Gauteng to investigate the ability of a hospital to deliver antiretroviral medication efficiently [62]. The study considered three input variables and four output variables over 14 public hospitals, with the hospitals being reorganised into 42 observations, as the data was originally structured as monthly. These observations were divided based on the size of the hospitals. The study concluded that small-scale medical facilities and medical facilities offering more technical medical services, such as surgeries, wasted fewer resources and hence delivered medical services more efficiently. The study considered both the CCR and the BCC models for comparison and analysis of results. The technique used by Ngoie [62] of making a distinction between the size of DMUs for comparability, and identifying the similarities within those distinctions is relevant to the investigation in this thesis.

Avkiran [3] investigated the technical and scale efficiencies of Australian universities using DEA. The three performance models that were considered was the overall performance, the performance on fee-paying enrolments and the performance on educational service delivery. The study reiterated the ability and the appropriateness of DEA evaluating multiple inputs and outputs at once. Avkiran used a different set of inputs and outputs for each performance model considered. All the models evaluated DEA with two inputs and three outputs, excluding the model of performance on fee-paying enrolments, which only considered two outputs. Both the CCR and the BCC models were applied at varying returns to scale. The study concluded that all the models performed well but that there was an opportunity to improve efficiency, and that there was a potential for more universities to downsize because they were operating at decreasing returns to scale. The final conclusion was that academic institutions like universities are obliged to apply efficiency analysis tools like DEA to improve productivity [3].

2.4 Efficiency measurement in the retail industry

DEA was developed primarily to determine the efficiency for non-profit organisations [17, 48]. The inputs and outputs involved in the productivity for such organisations were rarely affected by market information or external factors beyond the control of the organisation. Businesses and organisations that operate for profit, however, were also able to benefit from the non-parametric nature of DEA. Thus, DEA became of interest to firms³ and to other institutions looking to improve productivity in everyday business. This implies that there are indeterminable inputs and outputs regulated by changes in the market, competition, demographics, geography and other external factors that are difficult to measure, yet may have an intrinsic effect on efficiency which cannot be recorded and measured by DEA [82]. This leads to the belief that such inputs and outputs beyond the control of a firm are not regarded in the DEA model.

The retail industry is an industry where efficiency in a supply chain is quintessential. A majority of the relevant inputs and outputs needed for DEA are within the control of a retailer, and any opportunity to identify ways of reducing costs or maximising revenue is beneficial to any retailing entity. The selected inputs and outputs of any retailer will be relevant to the goals, objectives and financial situations of the retailer [80], and will allow for good governance and management

³The terms ‘firm’ and ‘industry’ implies an organisation or institution that operates for the purpose of generating and earning income [17].

of service units [82].

Donthu and Yoo [32] investigated the retail productivity for a chain of restaurants with 24 outlets using DEA, and expressed the importance of productivity relative to other service units rather than only absolute measurement of performance of service units in isolation. Absolute measures of productivity are ratio tests and formulas, and do not take relative performance, or performance within an environment, into account. The study thus focused on measuring efficiency at an individual store level. The study also considered other techniques that have been applied to retail productivity, such as regression, and comments on the subjectivity of weights from applying a cost function in regression when calculating indexes, as one functional form is imposed on all observations. The authors comment in the rich diagnostic information obtained when DEA is run at an individual outlet level. This is done by way of the efficiency reference set of DMUs that correspond to an inefficient unit. The limitations of traditional productivity measurement techniques mentioned in this study are the reason for applying DEA to this study. DEA creates a flexible piecewise linear function and does not force or impose a particular functional form on the data [32].

The study by Donthu and Yoo [32] is similar to the scope of this thesis, in that DMUs of the same chain are considered in this study, which is characterised as internal benchmarking. The study considered two outputs and four inputs for the DEA model. One of the outputs was financial and the other was behavioural, while three of the four inputs were controllable and one input was uncontrollable. Since DEA does not require that inputs and outputs are measured by the same units, this adds to the benefit of this model for measuring retail productivity.

Sharma and Choudhary [80] analysed the operational efficiency for a sample of 200 retail outlets for the Chandigarh Tricity over two years to determine if any relationship exists between efficiency and the size of stores. This is one of the larger sample sizes for DEA compared to most studies found in literature. Despite this, the study found no significant relationship between the efficiency scores obtained and the size of the retail stores. The findings also suggested that the percentage of stores operating efficiently in the Chandigarh Tricity was very low. This may be as a result of the larger calculation group size.

The authors also commented on the quality of the inputs and outputs used in the study, and suggested that the variables should reflect the retail firm's goals, objectives and sales situations. The study considered two outputs and three inputs, and these variables were financial and behavioural in nature. It was also suggested that profits and margins be considered as variables in the study, since there was no data to apply it. The number of efficient DMUs in this study was 30 out of the sample of 200 outlets [80].

A recent study was conducted by Gandhi *et al.* [38] which explored the production efficiency of 18 retailers in a growing Indian retail industry using the CCR and the BBC models. The CCR model identified five of the 18 retailers as performing efficiently, and the BCC model identified seven efficient DMUs. Two input variables and two output variables were identified for the DEA models, and were kept to a minimum to adhere to the convention of DEA, which is further explained in § 2.5, and for greater discriminatory power.

Similarly to the study by Donthu and Yoo [32], the study also commented on the rich diagnostic information obtained of DEA. One of the most important results from this study is that there is a large opportunity for improvement for the considered DMUs, with some units performing at an efficiency level as low as 12%. There is also great dispersion of the efficiency scores when the standard deviation of the sample is taken into account.

Gandhi *et al.* [39] conducted further research by looking at multiple benchmarking tools for generalised Indian retailers, with DEA being a prominent tool used, and then expanded the

study to include similar retailers across the world as well as benchmarking at a merchandise level. They also conducted a study of internal and external benchmarking of two generalised Indian retailers, and then benchmarked them with Wal-Mart. They found that there is merit in measuring productivity with competitors or other service units in the market. The objective of the study was to identify ways of improving efficiency without compromising the service quality [39].

2.5 Input and output mix

The selection and quality of decision-making units has great significance towards the result of DEA, since all DMUs are compared to one another in the DEA model and an efficiency score is calculated relative to the best performing DMU(s). The number of DMUs under consideration is also important, as a large number of observations offers better discriminatory power, because more DMUs adds to the linear program of DEA. This means that the model becomes computationally expensive as more DMUs are processed by the model [82], since DEA has to be solved for as many times as there are DMUs. The aim of this study is to use DEA for a much larger calculation group of DMUs than in previous studies for an increased level of discriminatory power and to analyse the effect it has on the efficiency of these units. The Retailer also has a unique distribution network, which warrants the need for this study.

The number of inputs and outputs also has an effect on the accuracy of DEA models. An increase in the number of inputs and outputs has, to some extent, a negative effect on the discriminatory power of the DEA models. The most crucial aspect of applying the DEA models is the specification of inputs and outputs [82]. Irrelevant inputs or outputs, or a misrepresentation of these components, will produce misleading results of efficient and inefficient DMUs. The process of just determining inputs and outputs can be valuable to a company, since this is not an activity that is done often. It is useful to identify what contributes to the performance of decision-making units, what influences productivity and which variables are considered to be relevant for the retailer.

There are several suggestions for the ideal number of DMUs to be considered from literature. Boussofiane *et al.* [12] uses a minimum of $(m \times s)$ DMUs, where m is the number of inputs and s is the number of outputs, while Golany and Roll [42] suggests that at least $2 \times (m + s)$ DMUs should be used. Bowlin [74] recommends at least $3 \times (m + s)$ DMUs, while Dyson *et al.* [33] prefers at least $(2 \times s \times m)$ DMUs when applying DEA. Vassiloglou and Giokas agree with Golany and Roll that the number of DMUs should be at least twice the sum of the number of inputs and outputs for DEA to perform more powerfully, and should be considered as a rule of thumb [90].

This thesis will consider the number of DMUs that at least satisfies all the suggested criteria from literature in order to satisfy the minimum requirements for discriminatory power, and also investigate the sensitivity of DEA at different sample sizes. This thesis will investigate samples that satisfy at least three times the sum of input and output variables, but will make concessions for samples that satisfy most of the criteria listed above. The size of the calculation groups in this thesis range from 12 to 1 207 DMUs for different grouping criteria. This will allow for commentary on the discriminatory power of the results.

2.6 The benefits of DEA

A benefit of DEA models is it does not require the units of inputs and/or outputs to be measured using the same unit measures. Inputs and outputs are free to vary in scale, whether it is monetary or physical units of measure [82]. The use of DEA has become popular when determining the efficiency of non-profit organisations for the non-parametric nature of the models [48]. This is because DEA models do not require any financial evaluations of any kind. Despite the fact that the model does not require financial information, DEA may be used in evaluating efficiencies for profit-seeking organisations [81], where financial evaluations may not be easy to determine either. DEA is a useful benchmarking tool because the model only requires activity information of an organisation. Information of this nature can be attained easily, which results in a low information cost to an organisation [48].

The benefits of DEA from a managerial perspective is that DEA has a conservative approach to calculating efficiencies of DMUs. The maximisation function of the DEA model enables the evaluated efficiency to be understated and giving each DMU “the benefit of the doubt” in making each unit as efficient as possible [82]. This bias enables entities to use DEA confidently, since any DMU that is identified as inefficient can be assured that it is definitely inefficient relative to other service units. Potential cost and resource savings are also determined by the DEA model, based on the relative performance of service units. This allows entities the opportunity to make changes to improve efficiency using inputs and/or outputs within their control, as identified by DEA models.

DEA is one of the few benchmarking tools that is able to determine efficiency using multiple inputs and/or multiple outputs. The weights assigned by the DEA model are calculated in order to maximise the efficiency of a considered DMU. Any other set of weights for the given set of DMUs (subject to all the inputs and outputs of the DEA model) would not allow for the maximum efficiency as is determined by DEA. The assigned weights are as efficient as possible, so any other set of weights competitive with the market compared to the weights from DEA may cause greater inefficiencies and the opportunity to improve inefficient units may be greater [82].

An additional characteristic of DEA models is that it identifies a group of DMUs against which each inefficient unit is found to be most directly inefficient. This set of DMUs is known as a peer group or an efficiency reference set (ERS)⁴. The ERS is calculated to identify the efficient DMUs which every inefficient unit is relatively inefficient to, i.e. the ERS of an inefficient DMU is a subgroup of efficient DMUs which directly identifies that DMU as inefficient. If a DMU has an efficiency score of 100%, it implies that this DMU is its own peer group [82].

The DEA model will also assign weights to each ERS member to determine the efficiency score. These weights identifies a mixture of the ERS that can be used to create a composite or virtual DMU that processes the same level of outputs as the inefficient DMU, but which requires less inputs. This composite DMU, made up of weights from efficient DMUs, provides a benchmark for the inefficient DMU to perform better. This also presents retailers with potential cost saving opportunities that are not identified by other benchmarking tools [82].

2.7 The shortcomings of DEA

Various extensions of the original DEA model have made it an extremely versatile and powerful tool in determining the efficiency of DMUs. However, the capabilities of DEA are limited. DEA

⁴The terminology “peer group” will be used interchangeably with “efficiency reference set” in this study.

is a benchmarking tool for relative efficiency. The model is not able to explicitly determine the optimal efficiency of DMUs, due to the limitations of technology and without changing the production process. The efficiency scores of DMUs cannot be directly compared with one another, as this score only reflects the spread of efficiencies within the sample and are not necessarily directly relative to the same ERS of DMUs [21].

The identification of units as inefficient are dependent on the mix of inputs and outputs of the model. The inputs and outputs under consideration will determine the criteria by which efficient and inefficient DMUs are identified. It is also important to consider that a mixture of monetary and physical measures of input and outputs will have an effect on whether DMUs are price efficient or not. DEA is able to identify which units are performing efficiently or not, but the nature of the efficiency is not clear when the mix of inputs and outputs is diverse. The omission of an important input or output can cause biases in the results. A large set of inputs and outputs and/or a small sample set of observed DMUs may result in many DMUs producing on the efficient frontier and decreasing the discriminatory power of the DEA models [21].

Although DEA has a conservative approach to maximising efficiencies, there is a risk that all DMUs under consideration are identified as efficient, which deems the model ineffective. The number of efficient DMUs tends to increase as the number of input and output variables increase. This also causes a loss in discriminatory power of the DEA model, since there are more inputs and outputs being considered, which means that there will be different combinations of inputs and outputs that may identify more DMUs as efficient. Ultimately, it remains the responsibility of the organisation to analyse the necessary activity changes in detail in order to perform efficiently in future [48].

Despite the shortcomings of the model, DEA is still considered as the best method for benchmarking. This is because there are no assumptions made on the distribution of the data, multiple inputs and multiple outputs can be considered at once, and DEA is able to identify the direction in which inefficient DMUs can improve productivity. DEA is also able to evaluate non-controllable variables that are free to vary in units.

CHAPTER 3

DEA models

Contents

3.1	The mathematical formulation of DEA	25
3.2	DEA efficient and weak efficient units	27
3.3	Constant returns to scale	28
3.3.1	Input-oriented frontier of CRS	30
3.3.2	Output-oriented frontier of CRS	32
3.4	Variable returns to scale	34
3.4.1	Input-oriented frontier of VRS	36
3.4.2	Output-oriented frontier of VRS	38
3.5	Technical and scale efficiency	40

The goal of the standard DEA model is to maximise the efficiency of a decision-making unit by calculating the best possible set of weights in order to achieve this efficiency. A decision-making unit that achieves an efficiency score of less than 100% has the potential to produce the same level of outputs with fewer inputs. The weights assigned by DEA are optimised for each input and output under evaluation.

The standard DEA model can be solved through various approaches. Each approach has significant differences in the way that DEA is performed and the results achieved. The approaches discussed in this chapter can be formulated in terms of input-orientation or output-orientation, and with different returns to scale. This chapter aims to present the mathematical formulations of the various forms of DEA and the input and output orientations of each form.

This chapter begins with a detailed description of the formulation of DEA, its dual and the calculation of the efficiency reference set in § 3.1. Strong and weakly efficient units as it pertains to the slack variables of DEA is described in § 3.2. The formulation of the CCR and BCC model, which considers different returns to scale, are described in detail in § 3.3 and § 3.4, respectively.

3.1 The mathematical formulation of DEA

The linear programming (LP) technique for solving the set of weights to achieve the maximum efficiency ratio of weighted outputs to weighted inputs for a single DMU, relative to n DMUs, was first formulated by Charnes *et al.* [17], and has since been the basis for formulating a DEA model. This formulation of a DEA model is known as the multiplier model¹ (also known as

¹The term is derived from the assigned weights, u_r and v_i , which represent the output and input multipliers, respectively.

the primal model) and uses equation (1.1) as the objective function. The parameters used to formulate the model are defined by letting

x_{ij} be the value of input i on DMU j , and
 y_{rj} be the value of output r on DMU j .

In the fractional form of the DEA model used to determine the efficiency score of DMU j_o (relative to the set of n DMUs), given as θ_{j_o} , the objective is to

$$\text{maximise} \quad \theta_{j_o} = \frac{\sum_{r=1}^s u_r y_{rj_o}}{\sum_{i=1}^m v_i x_{ij_o}} \quad (3.1)$$

subject to

$$\frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad j \in \{1, \dots, n\}, \quad (3.2)$$

$$u_r, v_i \geq 0, \quad i \in \{1, \dots, m\} \text{ and } r \in \{1, \dots, s\}, \quad (3.3)$$

where u_r is the weight assigned to output r and v_i is the weight assigned to input i . The variable weights are determined by solving the DEA model, which means that the data from all the DMUs are used as a reference set for determining the value of the weights. The DEA model calculates the efficiency score relative to other DMUs. This means that the efficiency scores of every DMU is used as a constraint in the LP. The maximisation function ensures that the most favourable weighting that the constraints will allow, will maximise the efficiency for the considered DMU. It is important to note that the weights are obtained directly from the observed data and is only subject to the constraints of the efficiency of other DMUs [17]. This ensures that no other set of weights will give a more favourable efficiency score relative to the reference set. The efficiencies of all units is determined by solving the fractional multiplier model and setting the objective function equal to each DMU.

The multiplier model in fractional form may be converted into algebraic form in order for the methods of linear programming to be applied more readily. The objective of the algebraic form of the multiplier model is to

$$\text{maximise} \quad \sum_{r=1}^s u_r y_{rj_o} \quad (3.4)$$

subject to

$$\sum_{i=1}^m v_i x_{ij_o} = 1, \quad (3.5)$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j \in \{1, \dots, n\}, \quad (3.6)$$

$$u_r, v_i \geq 0, \quad i \in \{1, \dots, m\} \text{ and } r \in \{1, \dots, s\}. \quad (3.7)$$

Objective function (3.1) has been linearised in objective function (3.4). This is to recognise that maximising the relative magnitudes of the numerator and the denominator are important, and not their actual values [12]. Hence, the numerator is maximised and the denominator has been set to one² in constraint (3.5).

The alternative to solving the multiplier model is to solve the enveloping³ (or dual) model. The multiplier model has $(s+m)$ variables, which means that the enveloping model will have $(s+m)$ constraints. Similarly, the multiplier model has $(s+m+n+1)$ constraints, which makes the computation of the multiplier model for n DMUs over m inputs and s outputs considerably larger and more time-consuming when compared to the enveloping model. The objective of the algebraic formulation for the enveloping model [73], is to

$$\text{minimise } \theta_{j_o} \quad (3.8)$$

subject to

$$\sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{ij_o}, \quad i \in \{1, \dots, m\}, \quad (3.9)$$

$$\sum_{j=1}^n \lambda_j y_{rj} \geq y_{rj_o}, \quad r \in \{1, \dots, s\}, \quad (3.10)$$

$$\lambda_j \geq 0, \quad j \in \{1, \dots, n\}, \quad (3.11)$$

where λ_j is the weight assigned to DMU j . Constraint set (3.9) ensures that the weighted sum of the inputs of all other DMUs is at most equal to the inputs of the DMU being considered and constraint set (3.10) ensures that the weighted sum of the outputs of the other DMUs is at least equal to that of the DMU being evaluated [82].

The enveloping model is not only better for computational convenience compared to the multiplier model, but it also provides additional information on the relative efficiencies determined by DEA. The enveloping model seeks the values of the weights of the DMUs, λ_j , to construct a composite unit to compare with the DMU under evaluation by identifying the outputs, $\sum \lambda_j y_{rj}$, $r \in \{1, \dots, s\}$, and inputs, $\sum \lambda_j x_{ij}$, $i \in \{1, \dots, m\}$, that outperform DMU j_o , if $\theta_{j_o} < 1$.

The units with non-zero λ_j values (i.e. the optimal values of λ_j) are units that form the peer group for the DMU under consideration, and provide targets for DMU j_o [12]. When the DEA model minimises θ_{j_o} and cannot find λ_j for any of the DMUs that will generate an efficiency below 100%, this implies that there is no opportunity to improve on the efficiency of DMU j_o when compared to the performance of other DMUs, and hence relative efficiency is achieved for DMU j_o [82].

3.2 DEA efficient and weak efficient units

The identification of an ERS to an inefficient unit is important, because some action must be taken to make that DMU efficient, which means identifying targets for such a DMU to achieve. The information from the ERS can specify where a reduction of inputs must occur in order to perform efficiently. These individual input reductions are known as input slacks.

²In the algebraic formulation of the multiplier model, the constant can be any value. This study will focus on setting the constant arbitrarily to 1.

³The term is derived from the inefficient DMUs being identified below the efficient frontier, hence the efficient service units “envelope” the inefficient DMUs.

Input and output slacks can be determined after calculating the enveloping DEA model [96]. The DEA slacks can be obtained once the enveloping model has been solved, where the objective is to

$$\text{maximise} \quad \sum_{i=1}^m S_i^- + \sum_{r=1}^s S_r^+ \quad (3.12)$$

subject to

$$\sum_{j=1}^n \lambda_j x_{ij} + S_i^- = \theta_{j_o}^* x_{ij_o}, \quad i \in \{1, \dots, m\}, \quad (3.13)$$

$$\sum_{j=1}^n \lambda_j y_{rj} - S_r^+ = y_{rj_o}, \quad r \in \{1, \dots, s\}, \quad (3.14)$$

$$\lambda_j \geq 0, \quad j \in \{1, \dots, n\}, \quad (3.15)$$

where $\theta_{j_o}^*$ is the DEA efficiency score obtained from objective function (3.8) of the enveloping model, and S_i^- and S_r^+ are the input and output slacks, respectively.

The complete calculation for the enveloping model thus involves two stages. The first stage determines the efficiency score, θ^* , and the second stage calculates the slack variables while keeping θ^* fixed. These two stages can be formulated into one LP to improve the convenience of computation. The performance of a DMU is fully efficient⁴, if and only if both

- (i) $\theta^* = 1$, and
- (ii) all slacks $S_i^- = S_r^+ = 0$.

The performance of a DMU with $\theta^* = 1$, but where all slacks $S_i^- \neq 0$ and/or $S_r^+ \neq 0$ for some i and r , is known as being weakly efficient [59]. In both cases, a DMU with $\theta = 1$ is considered technical efficient. If not equal to 1 (that is, a DMU that does not operate on its production function), then it is technical inefficient [36]. Technical efficiency investigates how well inputs are converted into outputs in the production process [3]. A DMU that is weakly efficient implies that it can still produce greater outputs given its current input level, or reduce its input usage given its current level of outputs [58], although it is operating at an efficiency score of $\theta^* = 1$. The weakly efficient DMUs and fully efficient DMUs will form the efficient frontier [59].

3.3 Constant returns to scale

Returns to scale (RTS) frontiers is the rate of substitution between inputs and outputs for each segment of the frontier. There are two types of RTS formulations of the DEA model. Charnes, Cooper and Rhodes [17] developed the CCR model, which makes use of constant returns to scale (CRS). This refers to the scenario where an increase of all input factors results in outputs increasing proportionally. This means that a k -fold change in inputs leads to a k -fold change in the outputs. For example, a DMU that receives 3 units of an item and sells 3 units has the same expected performance as a DMU that receives and sells 20 units. These DMUs are seen

⁴Full efficiency, also known as Pareto efficiency or DEA efficiency, is the state in which it is not possible to change the assignment of resources to improve the state of at least one individual, without worsening the state of at least one other individual [47].

as performing the best as they possibly can in reality, and hence they exhibit constant returns to scale.

The following example illustrates the different states of returns to scale. This example investigates the returns to scale of ten decision-making units. The assumption follows that only one input, x_1 , and one output, y_1 , is considered for convenience. The data of the values of each DMU are given in Table 3.1 and the data are plotted in Figure 3.1. The values of the output-to-input ratios for each of the observations in Table 3.1, where DMU_B has the highest ratio and is therefore the most efficient, and DMU_I has the lowest ratio and is thus the least efficient DMU out of the sample.

DMU	A	B	C	D	E	F	G	H	I	J
Input x_1	2	3	3	4	5	5	6	8	7.5	6
Output y_1	1	3	2	3	4	2	3	5	2	4.5
y_1 / x_1	0.5	1	0.667	0.75	0.8	0.4	0.5	0.625	0.267	0.75

TABLE 3.1: Data of ten DMUs given input x_1 and output y_1 .

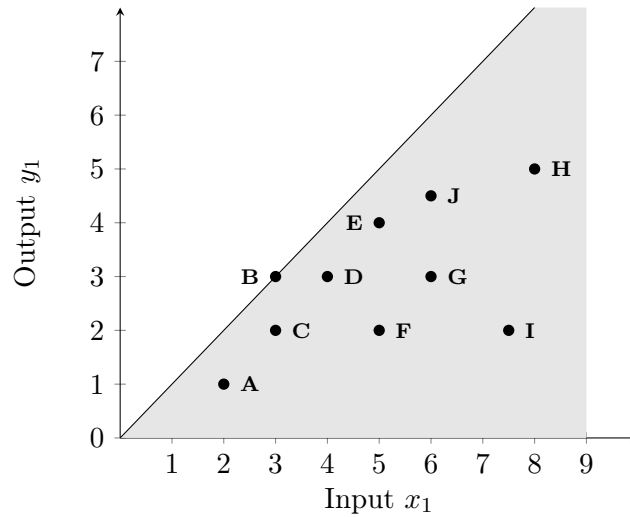
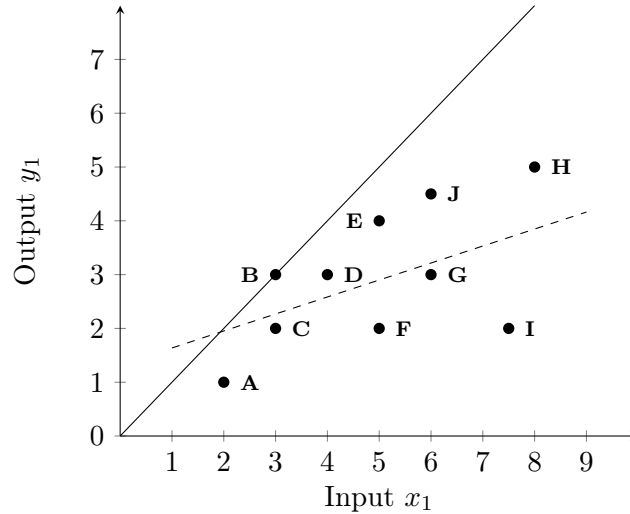


FIGURE 3.1: Efficient frontier of constant returns to scale.

Under constant returns to scale, the efficient frontier starts from the origin through the DMU with the best efficiency ratio. The DMU with the greatest output-to-input ratio from the example is DMU_B. The efficient frontier is thus extended, and becomes the reference of efficiency for all inefficient DMUs. The efficient frontier touches at least one point and all the other points will lie beneath this frontier. Thus, any DMU that is not on the efficient frontier and falls within the shaded region of Figure 3.1 will form the envelope of inefficient units relative to the DMU(s) that are situated on the efficient frontier.

For comparison, Figure 3.2 shows the data points from the data in Table 3.1 with the efficient frontier fitted to the data, represented by the solid line. In addition to the efficient frontier, a statistical regression line is also fitted to the data, as represented by the dotted line. The regression line is fitted through the “middle” of the data, as is typical of statistical regression, to show the central tendency of the data. Points that lie above the regression line is portrayed as being superior and points below the regression line as inferior or unsatisfactory. The performance of a point relative to the regression line is calculated as the magnitude of the deviation between that point and the regression line.

FIGURE 3.2: *Efficient frontier versus the regression line.*

There are fundamental differences between using regression analysis and DEA. The efficient frontier in DEA identifies a “best-practice” benchmark for other DMUs, while statistical regression benchmarks relative the average performance, and hence takes the averages of the best and worst performing DMUs as a basis for suggesting where improvements can be made [23].

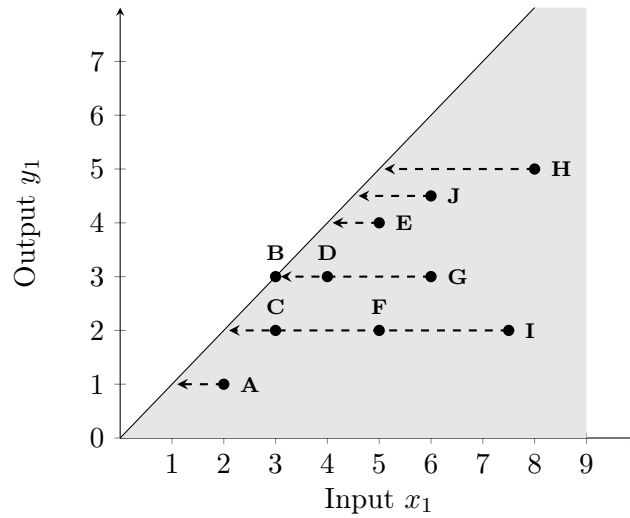
This example only considers the scenario where one input is weighted against one input. In the case where there are multiple inputs and/or outputs, the DEA model will identify the weights with which the inefficient units must adjust their inputs in order to achieve an efficient level of output, or how to maximise their output level without altering input levels. All DEA models can be viewed from two orientations when determining from which perspective an inefficient DMU can become efficient relative to the efficient frontier, namely the input-oriented and output-oriented models [96]. Input-oriented models evaluate whether a DMU can reduce its inputs, given that the current output levels remain constant. Output-oriented models are used when testing whether a DMU can increase its output levels, given that the level of input remains constant, relative to all other DMUs [96].

3.3.1 Input-oriented frontier of CRS

The objective of the input-oriented frontier approach under CRS is to reduce all inputs of DMUs performing inefficiently. This is to ensure that DMUs maintain the same level of output. Figure 3.3 shows the potential shift of inefficient DMUs to a state of efficiency once DEA has been applied from an input-oriented perspective under constant returns to scale. The dashed arrows in Figure 3.3 indicate that decreasing the inputs will result in these inefficient units producing at an efficient level of output.

Table 3.2 contains the efficiency scores, θ , of the DMUs under CRS, and the ERS of DMUs which each unit is directly inefficient to, which is DMU_B for all of the DMUs. This is because the efficient frontier is obtained by DMU_B being the best-practice DMU from the set of DMUs. The efficiency scores are obtained by substituting the values of the inputs and outputs contained in Table 3.1 into equations (3.8)–(3.11).

Consider DMU_D in Figure 3.3. DMU_D is producing the same level of output y_1 as DMU_B , but at a higher level of input x_1 . This means that there is an opportunity for DMU_D to reduce its input usage without compromising the performance of output. This is the case for all DMUs that do

FIGURE 3.3: *Efficient frontier of input-orientation under constant returns to scale.*

DMU	CCR (θ)	ERS
A	0.500	B
B	1.000	B
C	0.667	B
D	0.750	B
E	0.800	B
F	0.400	B
G	0.500	B
H	0.625	B
I	0.267	B
J	0.750	B

TABLE 3.2: *The efficiency scores and the ERS of ten DMUs under constant returns to scale.*

not lie on the efficient frontier. The efficiency score of DMU_D is 0.75, and it is evident that DMU_B is performing at the same level with less input. The efficiency score may be interpreted as DMU_D reducing its input usage to 75% of its current level, in order for it to be considered as efficient. Thus, by multiplying the efficiency score of DMU_D with its current input level, the value of the projected input is given by

$$0.75 \times (\text{Input of } DMU_D) = (\text{Input of } DMU_B). \quad (3.16)$$

This may be done for all inefficient DMUs to project their performance onto the efficient frontier. In the case of multiple inputs and/or outputs, the efficiency score may be applied in the same way to project by multiplying the efficiency score of a DMU with its input variable values to determine the input values projected on to the efficient frontier.

The LP for the input-oriented model is obtained by combining the LP in equations (3.8)–(3.11) with the slack LP formulation in equations (3.12)–(3.15), such that the objective is to

$$\text{minimise } \theta - \varepsilon \left(\sum_{i=1}^m S_i^- + \sum_{r=1}^s S_r^+ \right) \quad (3.17)$$

subject to

$$\sum_{j=1}^n \lambda_j x_{ij} + S_i^- = \theta x_{ijo}, \quad i \in \{1, \dots, m\}, \quad (3.18)$$

$$\sum_{j=1}^n \lambda_j y_{rj} - S_r^+ = y_{rjo}, \quad r \in \{1, \dots, s\}, \quad (3.19)$$

$$\lambda_j \geq 0, \quad j \in \{1, \dots, n\}, \quad (3.20)$$

where ε is a small non-Archimedean number, smaller than any positive real number ($0 < \varepsilon \ll 1$) and S_i^- and S_r^+ are the input and output slacks, respectively. The presence of ε in the objective function allows for the optimisation of the slack variables in the minimisation of θ . This model can also be considered a two-stage model where θ^* is first calculated by ignoring the slacks, and then movement on the efficient frontier is achieved by optimising the slack variables and fixing the θ^* in the following linear programming model.

For inefficient and weakly efficient decision-making units, there are efficiency paths for each DMU to follow in order to reach the efficient frontier. The formulae of the projected input and output values for inefficient and weakly efficient DMUs in the input-oriented model are

$$\hat{x}_{ijo} = \theta^* x_{ijo} - S_i^- = \sum_{j=1}^n \lambda_j x_{ij}, \quad i \in \{1, \dots, m\}, \text{ and} \quad (3.21)$$

$$\hat{y}_{rjo} = y_{rjo} + S_r^+ = \sum_{j=1}^n \lambda_j y_{rj}, \quad r \in \{1, \dots, s\}, \quad (3.22)$$

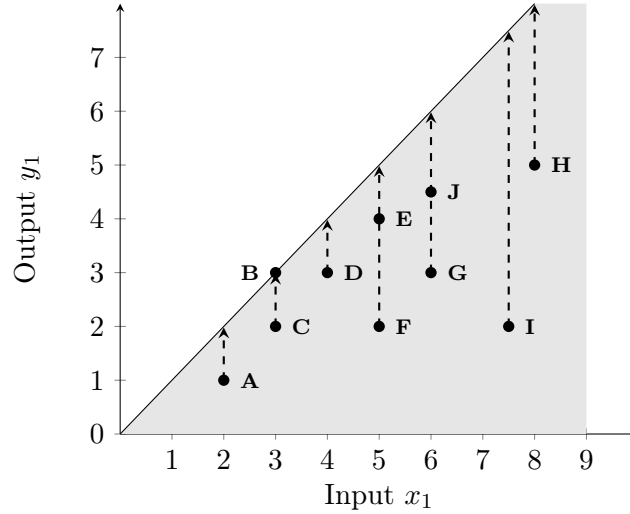
where θ^* is the efficiency score obtained from the enveloping model in equations (3.17)–(3.20), and S_i^- and S_r^+ are the input and output slacks, respectively. Equations (3.21) and (3.22) provide an efficient target for a specific inefficient DMU, and may be seen as a general formula for the calculation in equation (3.16) for multiple inputs and/or outputs.

3.3.2 Output-oriented frontier of CRS

Similar to the input-oriented model, the objective of the output-oriented frontier approach under constant returns to scale is to optimise the output level of DMUs performing inefficiently without influencing the level of inputs used in order to reach the efficient frontier. The efficiency scores of the DMUs in the output-oriented model remain the same as in Table 3.2. The orientation of the model does not affect the efficient frontier itself, and since the efficient frontier exhibits CRS, the same efficiency scores apply. Figure 3.4 shows the potential shift of inefficient DMUs to a state of efficiency once DEA has been applied from an output-oriented perspective under constant returns to scale. The dashed arrows in Figure 3.4 indicate that the rise of the output levels while the input levels remain unchanged.

Consider DMU_C in Figure 3.4. DMU_C is using the same amount of input x_1 as DMU_B, but is not producing the same level of output y_1 . This means that there is an opportunity for DMU_C to produce more of its output compromising the input level. Once again, this is the case for all DMUs that do not lie on the efficient frontier. The efficiency score for DMU_C given in Table 3.2 is 0.667. This may be interpreted as DMU_C performing at only at 66.67% of its potential. Therefore, in order to determine the new output value, the efficiency score may be divided from the current output level, such that

$$\frac{(\text{Output of DMU}_C)}{0.667} = (\text{Output of DMU}_B). \quad (3.23)$$

FIGURE 3.4: *Efficient frontier of output-orientation under constant returns to scale.*

This is evident in Figure 3.4, where DMU_B and DMU_C are both operating at the same level of input, but DMU_B is producing a greater output. Therefore, DMU_C has the potential to perform as well as DMU_B , as DMU_B is producing 1.5 times more output than DMU_C while using the same level of input.

It is important to note that the simplified example only considers a single input producing a single output [73]. The DEA model will identify what the optimal output level can be, should the input level remain unchanged for multiple inputs and/or multiple outputs, as described in § 1.6. The efficiency scores for the DMUs may be obtained with the same formulation as for the input-oriented model, since the efficiency scores are the same under CRS. Therefore, the efficiency scores are given in Table 3.2. The formulation of the LP in an output-oriented model differs from the input-oriented model in equations (3.17)–(3.20), but the efficiency scores remain the same. The objective of the output-oriented method, in terms of the enveloping model, is to

$$\text{maximise } \phi + \varepsilon \left(\sum_{i=1}^m S_i^- + \sum_{r=1}^s S_r^+ \right) \quad (3.24)$$

subject to

$$\sum_{j=1}^n \lambda_j x_{ij} + S_i^- = x_{ijo}, \quad i \in \{1, \dots, m\}, \quad (3.25)$$

$$\sum_{j=1}^n \lambda_j y_{rj} - S_r^+ = \phi y_{rjo}, \quad r \in \{1, \dots, s\}, \quad (3.26)$$

$$\lambda_j \geq 0, \quad j \in \{1, \dots, n\}, \quad (3.27)$$

where ε is a small positive number, S_i^- and S_r^+ are the input and output slacks, respectively, and where ϕ^* is the efficiency score of the output-oriented model. In the case of constant returns to scale, the value of ϕ in the output-oriented model is the same as the value of θ in the input-oriented model. The projected input and output values for inefficient and weakly efficient DMUs

to follow in order to reach the efficient frontier in the output-oriented model are formulated as

$$\hat{x}_{ij_o} = x_{ij_o} - S_i^- = \sum_{j=1}^n \lambda_j x_{ij}, \quad i \in \{1, \dots, m\}, \text{ and} \quad (3.28)$$

$$\hat{y}_{rj_o} = \phi^* y_{rj_o} + S_r^+ = \sum_{j=1}^n \lambda_j y_{rj}, \quad r \in \{1, \dots, s\}, \quad (3.29)$$

Equations (3.28) and (3.29) provide an efficient target for a specific inefficient DMU, and may be seen as a general formula for the calculation in equation (3.23) for multiple inputs and/or outputs.

3.4 Variable returns to scale

The BCC model, developed by Banker, Charnes and Cooper [6] in 1984, introduced an adaptation in the formulation of the CCR model to account for variable returns to scale in the DEA model. Variable returns to scale (VRS), unlike CRS, considers that DMUs could have different productivities and still be considered efficient at different returns to scale. This means that VRS takes into account the different efficiencies of DMUs performing under different situations, like varying production technologies. Variable returns to scale is the notion that an increase of all input factors may not necessarily result in a proportional increase of outputs [7, 65]. The efficient frontier, when VRS is considered, is concave, which allows the DEA model to accommodate more DMUs on the frontier [50]. The sensitivity of the DEA model to VRS greatly impacts the efficiency scores for all DMUs, regardless of the size of the sample of DMUs [40, 51].

Efficiency is expressed as the weighted sum of outputs over the weighted sum of inputs. When the weighted sum of outputs is equal to the weighted sum of inputs (i.e. when the variation in inputs results in an equal variation in the level of outputs), then production performs under CRS. When the weighted sum of outputs is greater than the weighted sum of inputs (i.e. when the variation of inputs is smaller than the variation in outputs), production is considered to have a non-decreasing return to scale (NDRS), or increasing returns to scale. When the weighted sum of outputs is less than the weighted sum of inputs (i.e. when the variation of inputs is greater than the variation in outputs), production is considered to have a non-increasing return to scale (NIRS), or decreasing returns to scale [7].

The CRS model overestimates technical efficiency when projecting an inefficient DMU to an efficient benchmark that is characterised by either NIRS or NDRS. If the efficient frontier exhibits CRS, then technical efficiency is not overestimated, but if production exhibits VRS, then the BCC model will benchmark technical efficiency [73].

The following example is a continuation of the example adapted from Cooper [23] to illustrate the different states of returns to scale. This example follows from the data in Table 3.1, again with the assumption that only one input, x_1 , and one output, y_1 , is being considered. The data of the values of each DMU are given in Table 3.1.

Under VRS, the efficient frontier is determined by all efficient DMUs identified by DEA. This implies that DMUs with an efficiency score of 1 will all form the efficient frontier. The efficient frontier is thus extended to form the envelope (the shaded region) as illustrated in Figure 3.5. The dotted line illustrates where the efficient frontier would be under CRS. Thus, the DMUs that are in the shaded region (DMU_C, DMU_D, DMU_F, DMU_G and DMU_I) lie within the envelope of inefficient units relative to the DMUs that are situated on the efficient frontier. This also means

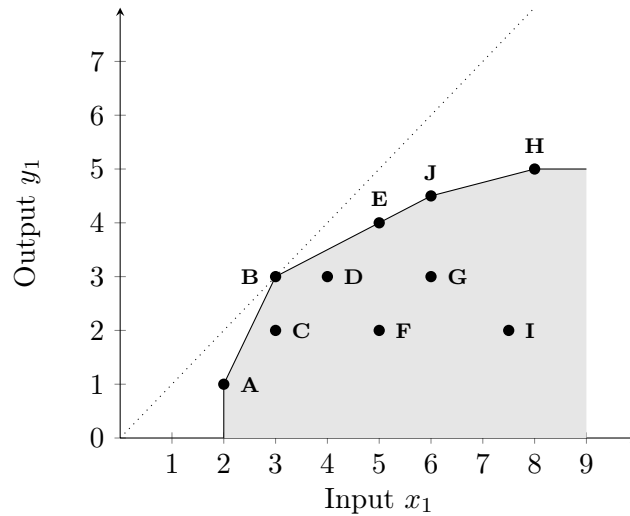


FIGURE 3.5: *Efficient frontier of VRS with all returns to scale.*

that performing technically inefficient relative to the efficient frontier under VRS and also scale inefficient relative to the efficient frontier under CRS [38]. DMUs that performed efficiently under CRS are also performing efficiently under VRS, which is DMU_B from the example, and may be seen in Figure 3.5.

Production is characterised as non-increasing return to scale (NIRS) when the variation of inputs is greater than the variation in outputs. This means that a greater consumption of inputs is required for a smaller increase in the level of output when compared to CRS. Figure 3.6 shows the efficient frontier under VRS when considering only NIRS. When the variation of inputs is smaller than the variation in outputs, production is characterised by non-decreasing return to scale (NDRS). This means that a greater level of output is produced from a smaller consumption of inputs when compared to CRS. Figure 3.7 shows the efficient frontier when only NDRS is taken into account [7].

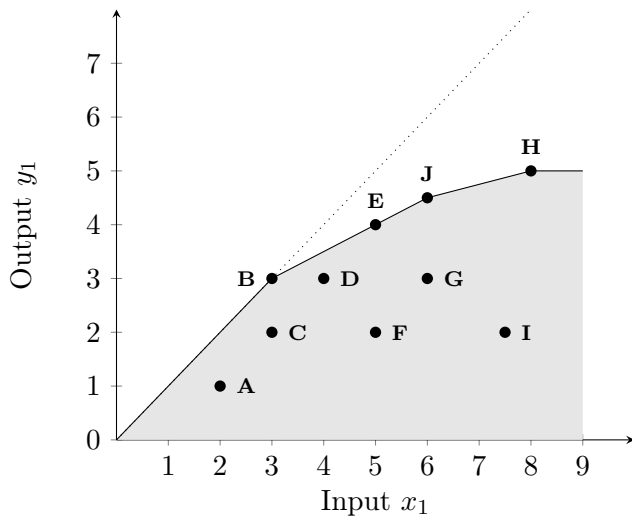


FIGURE 3.6: *Efficient frontier of NIRS.*

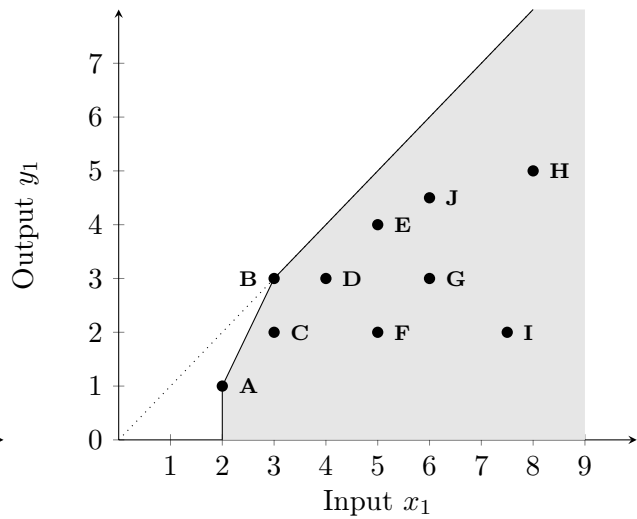


FIGURE 3.7: *Efficient frontier of NDRS.*

The formulation of the input-oriented method in terms of the enveloping model under variable returns to scale is the same as the formulation for DEA under CRS, characterised by

equations (3.17)–(3.20), with an additional constraint, which is $\sum_{j=1}^n \lambda_j = 1$. This constraint considers all VRS, and can be adjusted to consider the case where NIRS is considered (i.e. $\sum_{j=1}^n \lambda_j \leq 1$) or where NDRS is considered (i.e. $\sum_{j=1}^n \lambda_j \geq 1$).

3.4.1 Input-oriented frontier of VRS

Similar to the input-oriented model under CRS, the objective of the input-oriented frontier approach under VRS is to reduce all inputs of DMUs performing inefficiently in order to reach the level of output set by the efficient frontier. However, the efficient frontier is dependent on whether the model looks at non-increasing returns to scale, non-decreasing returns to scale or all variable returns to scale. For the purpose of this thesis, all VRS will be considered.

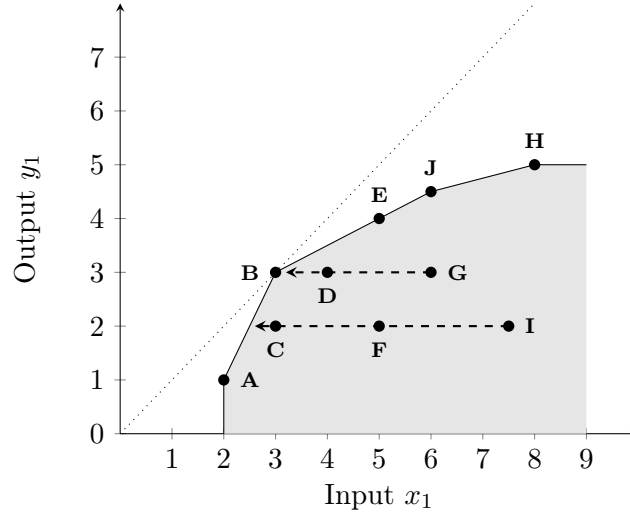


FIGURE 3.8: *Efficient frontier of input-orientation under variable returns to scale.*

Figure 3.8 shows the potential shift of inefficient DMUs to a state of efficiency under all returns to scale. Consider DMU_C in Figure 3.8, where the dashed arrow from DMU_C to the efficient frontier shows the decrease in input variables that is necessary to become efficient. Unlike the CRS model, DMU_C, for example, does not have to decrease its use of inputs as much as would be necessary if the efficient frontier of CRS is considered, since the line segment of the efficient frontier in Figure 3.8 which DMU_C is directly relatively inefficient to, is characterised by NDRS. This results in the peer group of DMU_C being characterised by efficient DMU_A and DMU_B.

DMU	BCC (θ)	ERS
A	1.000	A
B	1.000	B
C	0.833	A (0.5), B (0.5)
D	0.750	B
E	1.000	E
F	0.500	A (0.5), B (0.5)
G	0.500	B
H	1.000	H
I	0.333	A (0.5), B (0.5)
J	1.000	J

TABLE 3.3: *The efficiency scores and the ERS of input-orientation under variable returns to scale.*

The efficiency scores for the input-oriented VRS model and the ERS for each DMU is given in Table 3.3. If an ERS contains only one reference DMU j , the weight of DMU j on the considered DMU is $\lambda_j = 1$, and in cases where there is more than one DMU in an ERS, a weight (λ_j) is assigned to each DMU in the ERS. It is worth noting that there are more DMUs on the efficient frontier that were not considered to be efficient under CRS. This is due to the addition of the convexity constraint to the equations (3.17)–(3.20). It is also noted that efficient DMUs are their own peer group.

Consider a DMU_K, where the input and output variables for this DMU is $(x_1, y_1) = (4, 0.5)$, as shown in Figure 3.9. Figure 3.9 continues to consider DMUs A to F from the data in Table 3.1. In an input-oriented model, DMU_K will reduce its inputs to reach a point (say, point K') on the efficient frontier, without changing its level of output. This indicated by the dashed arrow in Figure 3.9.

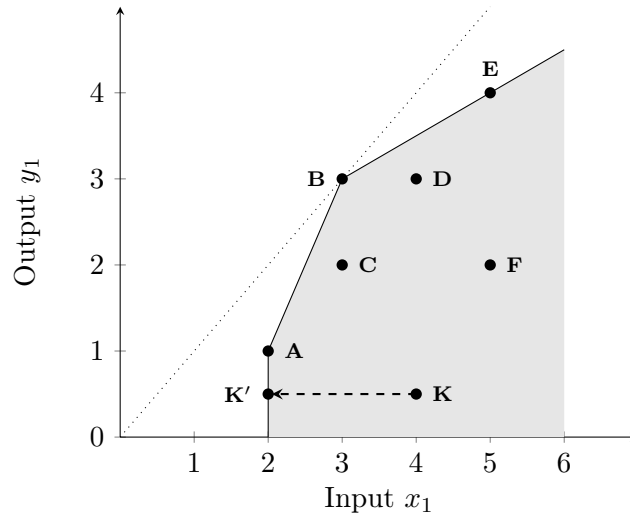


FIGURE 3.9: Slack variables with regards to the efficient frontier of VRS.

If DMU_K were to reach the frontier at point K', then DMU_K would be efficient. However, from Figure 3.9, it is evident that at that input level, it is possible to produce a greater level of output. This is because DMU_A is producing a greater level of output relative to the point K'. A DMU at point K' would thus be weakly efficient, since the efficiency score would be equal to 1, but there is a nonzero slack of output 1 being $S_1^+ = 0.5$. The slack variables are often not considered in DEA models under CRS when weak efficiency is concerned, since the efficient frontier is proportional and does not vary in scale in the way that the BCC model does, as a result of the additional constraint to the LP.

The efficiency scores of the DMUs under VRS for the input-oriented model in Table 3.3 are obtained by substituting the input and output values into the LP where the objective is to

$$\text{minimise } \theta - \varepsilon \left(\sum_{i=1}^m S_i^- + \sum_{r=1}^s S_r^+ \right) \quad (3.30)$$

subject to

$$\sum_{j=1}^n \lambda_j x_{ij} + S_i^- = \theta x_{ijo}, \quad i \in \{1, \dots, m\}, \quad (3.31)$$

$$\sum_{j=1}^n \lambda_j y_{rj} - S_r^+ = y_{rjo}, \quad r \in \{1, \dots, s\}, \quad (3.32)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (3.33)$$

$$\lambda_j \geq 0, \quad j \in \{1, \dots, n\}, \quad (3.34)$$

where ε is a small positive number, and S_i^- and S_r^+ are the input and output slacks, respectively. The efficiency score for DMU_C is 0.833, and this DMU is directly inefficient to DMU_A and DMU_B. The efficiency score may be interpreted as DMU_C reducing its input usage to 83.33% of its current level, in order for it to be considered as efficient. Thus, by multiplying the efficiency score of DMU_C with its current input level, the value of the projected input in terms of its ERS (with the relevant λ -values) is given by

$$0.833 \times (\text{Input of DMU}_C) = 0.5 \times (\text{Input of DMU}_A) + 0.5 \times (\text{Input of DMU}_B). \quad (3.35)$$

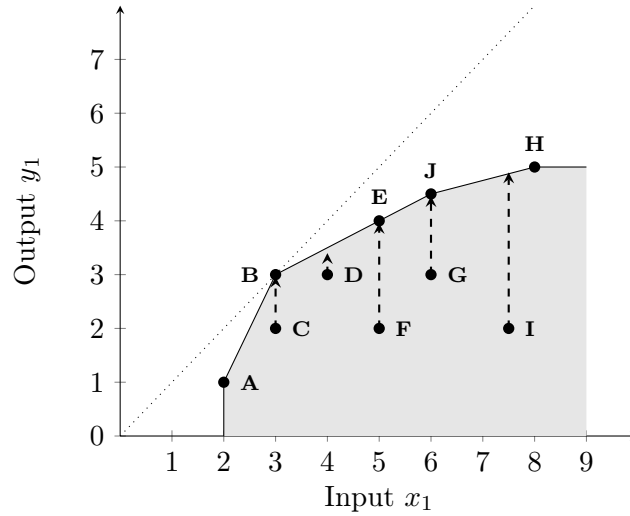
The formulae of the projected input and output values for inefficient and weakly efficient DMUs in the input-oriented model are consistent with equations (3.21) and (3.22), with S_i^- and S_r^+ are the input and output slacks, respectively, but the efficiency scores are obtained from the enveloping model in equations (3.30)–(3.34). Equations (3.21) and (3.22) may be seen as the general formula for the calculation in equation (3.35).

3.4.2 Output-oriented frontier of VRS

The objective of the output-oriented frontier approach under VRS, similar to the output-oriented frontier approach under CRS, is to optimise the output level of DMUs performing inefficiently without influencing the level of inputs produced in order to reach the efficient frontier. Figure 3.10 shows the potential shift of inefficient DMUs to a state of efficiency once DEA has been applied from an output-oriented perspective under variable returns to scale. The dashed arrows in Figure 3.10 indicates the rise of the output levels while the input levels remain unchanged. DMUs performing under NIRS do not need to increase its outputs as much as DMUs under CRS, as can be shown by the inefficient DMUs operating under the efficient frontier (namely, DMU_C, DMU_D, DMU_F, DMU_G and DMU_I) in Figure 3.10.

The formulation of the output-oriented method in terms of the enveloping model is similar to the formulation characterised by equations (3.24)–(3.27), with an additional constraint, which is $\sum_{j=1}^n \lambda_j = 1$. This constraint considers all VRS, and can be adjusted to consider the case where NIRS is considered (i.e. $\sum_{j=1}^n \lambda_j \leq 1$) or where NDRS is considered (i.e. $\sum_{j=1}^n \lambda_j \geq 1$). The scores of inefficient DMUs is calculated relative to the efficient frontier under varying returns to scale, and as a result, the relationship between the input-oriented and output-oriented efficiency scores is not equivalent under VRS as it was under CRS. The objective of the output-oriented model under VRS is to

$$\text{maximise } \phi + \varepsilon \left(\sum_{i=1}^m S_i^- + \sum_{r=1}^s S_r^+ \right) \quad (3.36)$$

FIGURE 3.10: *Efficient frontier of output-orientation under variable returns to scale.*

subject to

$$\sum_{j=1}^n \lambda_j x_{ij} + S_i^- = x_{ijo}, \quad i \in \{1, \dots, m\}, \quad (3.37)$$

$$\sum_{j=1}^n \lambda_j y_{rj} - S_r^+ = \phi y_{rjo}, \quad r \in \{1, \dots, s\}, \quad (3.38)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad (3.39)$$

$$\lambda_j \geq 0, \quad j \in \{1, \dots, n\}, \quad (3.40)$$

where ε is a small positive number, ϕ represents the output-oriented efficiency score, and S_i^- and S_r^+ are the input and output slacks, respectively. Constraint (3.39) is given as $\sum_{j=1}^n \lambda_j = 1$ to consider all VRS. Alternatively, $\sum_{j=1}^n \lambda_j \leq 1$ or $\sum_{j=1}^n \lambda_j \geq 1$ may be used when considering only NIRS or only NDRS, respectively.

DMU	BCC (ϕ)	ERS
A	1.000	A
B	1.000	B
C	0.667	B
D	0.857	B (0.5), E (0.5)
E	1.000	E
F	0.500	E
G	0.667	J
H	1.000	H
I	0.410	H (0.8), J (0.2)
J	1.000	J

TABLE 3.4: *The efficiency scores and the ERS of output-orientation under variable returns to scale.*

Consider DMU_I, with an efficiency score of 0.410 as determined by solving equations (3.36)–(3.40) and is given in Table 3.4. DMU_I is directly inefficient to DMU_J and DMU_H, as these units make up the efficient frontier for DMU_I. It is important to note that, unlike the efficiency

scores under CRS, the efficiency scores for the input and output oriented models are not the same, since the efficient frontier is subject to varying returns to scale. In order for DMU_D to perform efficiently relative to the other considered DMUs, the output of DMU_I is calculated as

$$\frac{(\text{Output of } DMU_I)}{0.410} = 0.8 \times (\text{Output of } DMU_H) + 0.2 \times (\text{Output of } DMU_J). \quad (3.41)$$

The efficiency paths for inefficient and weakly efficient DMUs in the output-oriented model are the same as equations (3.28) and (3.29), where ϕ^* is the output-oriented efficiency score calculated by DEA under VRS, and S_i^- and S_r^+ are the input and output slacks, respectively.

3.5 Technical and scale efficiency

Technical efficiency investigates how well inputs are converted into outputs in the production process [3]. DMUs that are technical efficient are performing efficiently with $\theta^* = 1$. Scale efficiency is also a concept that must be taken into consideration. Scale efficiency is the component that addresses the optimal level of volume activity, and acknowledges that a change in size of operations of a DMU will have an effect of the efficiency of a DMU [82]. It is also known as the technically efficient DMU under VRS [60], because of the difference in the return to scale used in CRS as opposed to VRS. This means that efficient DMUs under CRS are scale efficient and technically efficient, whereas efficient DMUs under VRS are technically efficient, but not necessarily scale efficient [50]. Technical and scale efficiencies are illustrated in Figure 3.11.

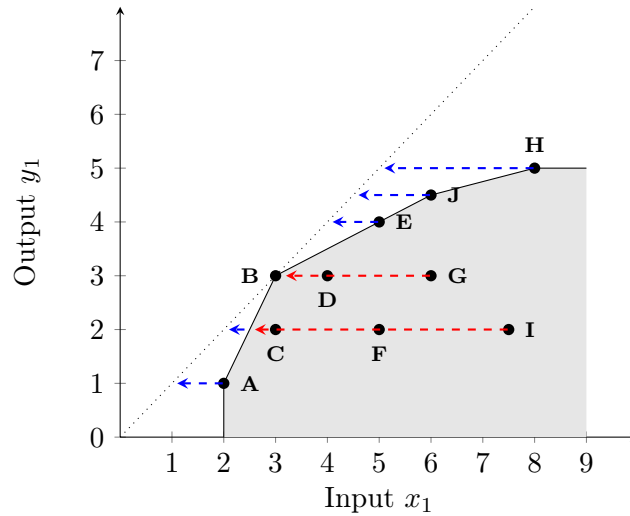


FIGURE 3.11: *Technical efficiency and scale efficiency under constant and variable returns to scale.*

Scale efficiency is the component that addresses the optimal level of volume activity, and acknowledges that a change in size of operations of a DMU will have an effect of the efficiency of a DMU [82]. It is also known as the technically efficient DMU of the BCC model [60], because of the difference in the return to scale used in the BCC model as opposed to the CCR model. This means that efficient DMUs the CCR model are scale efficient and technically efficient, whereas efficient DMUs in the BCC model are technically efficient, but not necessarily scale efficient [50].

The solid line in Figure 3.11 represents the efficient frontier of VRS, and the dotted frontier through the point B represents the efficient frontier of CRS. Technical inefficiencies in Figure 3.11 are illustrated by the dotted red arrows. DMU_B has the maximum output-input ratio, which

means that DMU_B is both technical efficient and scale efficient. When the efficient frontier of VRS is considered, DMU_A , DMU_E , DMU_J and DMU_H are technical efficient relative to the frontier, but are scale inefficient relative to the efficient frontier under CRS, as represented by the dashed blue arrows. DMU_C , DMU_D , DMU_F , DMU_G and DMU_I , which do not lie on any efficient frontier, are both technical inefficient and scale inefficient [26].

It is important to note that there are cases where an efficient DMU under VRS in the BCC model will be inefficient under CRS in the CCR model because of scale. Once again, the trade-off between using one model or the other depends on the production capabilities of the Retailer. An efficient DMU under CRS will remain efficient when VRS is considered, which means that there is certainty surrounding those efficient units. However, the changes in inputs and outputs are often not proportional, and VRS accounts for firms that produce at varying production capacities.

CHAPTER 4

Data validation and analysis

Contents

4.1	Introduction to the dataset	44
4.1.1	<i>Data attributes</i>	44
4.1.2	<i>Store attributes</i>	45
4.1.3	<i>The dataset</i>	46
4.2	Data validation	47
4.2.1	<i>Data entries</i>	48
4.2.2	<i>Duplicates of data entries</i>	48
4.2.3	<i>Negative entries</i>	48
4.2.4	<i>Disclosure of weeks</i>	48
4.2.5	<i>Uniqueness of store names</i>	48
4.2.6	<i>Flow of inventory</i>	49
4.3	Data analysis	49
4.3.1	<i>Base exclusive values</i>	49
4.3.2	<i>Opening and closing stock</i>	49
4.3.3	<i>Inflow quantity</i>	50
4.3.4	<i>Sales</i>	50
4.3.5	<i>Style code</i>	51
4.3.6	<i>Stores sales plan value</i>	52
4.3.7	<i>Service level value</i>	52
4.3.8	<i>Last day of the week (LDOW)</i>	52

The Retailer has a system that records all the sales information, from inventory on hand to the actual point of sale. The accuracy of the DEA model requires extensive real data for the performance of DMUs to be determined as accurately as possible. Data play a vital role in the generation of good results and for analysis. Intensive data validation and analysis is thus required to ensure data integrity. The scope of this study is based on all stores and all subclasses over different seasons. All data analysis and alterations were executed in SAS¹, the software used for data handling and analysis.

This chapter begins with an introduction to the dataset that is used in this study in § 4.1, which includes important attributes of the Retailer's data. Data validation follows in § 4.2 with information about how the data is prepared and validated. The chapter concludes with an analysis of the relevant data fields used to determine inputs and outputs in § 4.3.

¹SAS is powerful business analytics and business intelligence software. It is an integrated system of software solutions for the execution of various projects ranging from data entry and management to statistical and mathematical analysis [76].

4.1 Introduction to the dataset

The data consist of raw sales data and inventory levels summarised up to a weekly level. The data thus consist of opening and closing stock, as well as any flow of products to and from a store during a given week per subclass per style season. The data also contain detailed information on the nature of the product, like whether it is a replenishment or fashion item and whether it is promotional or not. The data have been captured from 28 January 2017 to 22 July 2017, and contains all information of SKUs sold during this period.

One main dataset is used throughout the study. All of the subclasses in this study are a subset of the baby boys' outerwear department. This department contains a collection of both summer and winter products, as well as products that are sold throughout the year. SKUs sold primarily in summer are denoted as being of the season SXX of the year 20XX, and similarly, winter products are denoted as being of the season WXX. This study will focus on W17, W16 and replenishment products (denoted with the season code 00), which make up a combined 84.59% of the data entries.

Region name	Branch No.	Subclass No.	Style code	Product type	Merchandise type	Style season code	LDOW
GAUTENG	8793	5875	3	FASHION	YEARLY	W17	28JAN2017
EMFULENI	6052	180	2	FASHION	YEARLY	W17	10JUN2017
CEDERBERG	6146	177	5	REPLENISHMENT	YEARLY	00	13MAY2017
BOTSWANA	549	8123	4	FASHION	WINTER	W17	27MAY2017
KWENA	989	188	3	FASHION	WINTER	W16	04FEB2017

TABLE 4.1: An example of the unique data entries of a store.

The entries in the data are unique by various criteria. The criteria are shown in Table 4.1, which shows entries of stores in the data containing information about that store and its unique identifiers, such as the **branch number**, the **subclass number**, its **season style code** and the **last day of the week** (LDOW). The combination of these four criteria, or data fields, make it possible to refer to a unique instance in the data; no two data entries will have the same information over all four these data fields.

4.1.1 Data attributes

The data contain sales information on all 1 207 stores over various regions in the country as well as abroad. Each store stocks a number of different products and subclasses of products which come in different styles. Table 4.2 contains the amount of information available for the seasons being considered, namely the number of stores that stock products from that season, the number of subclasses that have products from that season and the number of entries in the data relating to that specific season.

Description	W17	W16	00
Number of stores	1 201	1 195	1 201
Number of subclasses	44	39	12
Number of entries	976 430	653 940	265 739

TABLE 4.2: Attributes of seasons W17, W16 and 00.

It is important to note that products within a specific season are independent from one another, which means that products of W16 are not included in products of W17. They are products from entirely separate and unique seasons, although they may come from the same subclass.

The season represents the initial time that styles of that subclass were first sold to customers. Not all stores receive every style of product, and the products that a store receives is different every season.

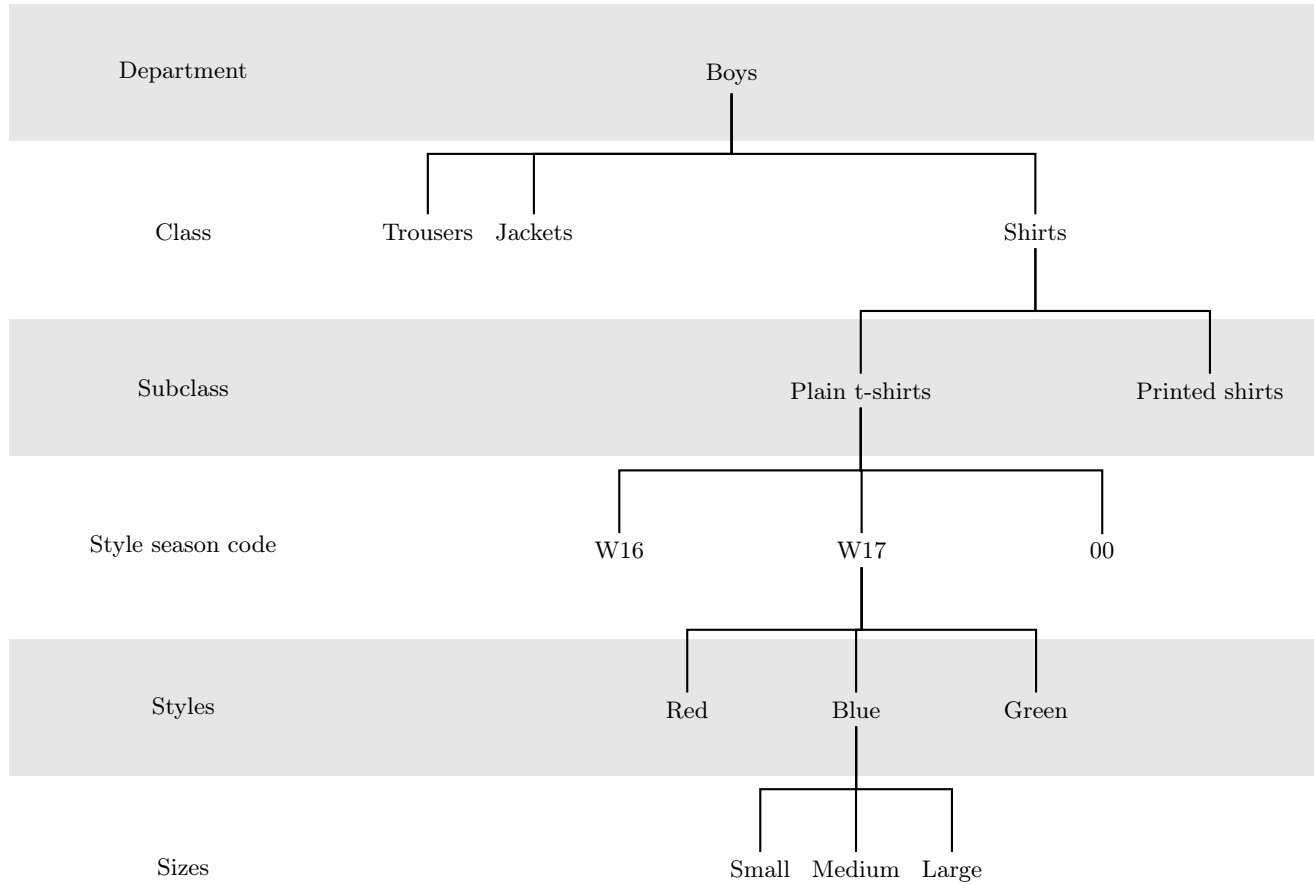


FIGURE 4.1: The merchandise hierarchy of product classification for the considered retailer.

Figure 4.1 shows a general view of the structure of the merchandise hierarchy for the Retailer. The merchandise hierarchy of the Retailer follows the same structure as retailers in literature [44], even as the assortment of products for different markets varies. The different departments focus on various product attributes in order to appeal to a broader target market.

The width of a retailer is determined by the departments, which can be divided into different categories, or classes. The subclasses of the Retailer proposes the breadth of product assortment, and the different product variants, which are the styles and sizes of the products, is known as the width [44]. A SKU is uniquely identified as belonging to a specific product classification criterion. The dataset consists of 28 seasons of products, and there are 12 classes of products containing 64 subclasses and that spans over 1 207 stores.

4.1.2 Store attributes

The Retailer has stores all across southern Africa. The stores have been divided into regions and the percentages of the total number of stores per region is given in Table 4.3. The regions

are listed in ascending order of the number of stores within that region. All the stores that sell products from season W17, W16 or 00 between 28 January 2017 and 22 July 2017 are considered. Stores are not only geographically separated, but also by the products they stock, known as divisions. Only stores that stock products from the baby boys' outerwear department are considered.

Region	Number of stores	Percentage
Swaziland	20	1.66%
Southern Namibia	43	3.56%
Northern Namibia	54	4.47%
Botswana	69	5.71%
Cederberg	85	7.04%
Emfuleni	86	7.13%
Kwena	86	7.13%
Langeberg	86	7.13%
North West	88	7.29%
Free State	91	7.54%
Lesedi	91	7.54%
Gauteng	98	8.12%
Limpopo	98	8.12%
Thekwini	105	8.70%
Tugela	107	8.86%
	1 207	100%

TABLE 4.3: *The percentage of stores of the Retailer located in 15 regions over southern Africa.*

The Retailer also makes a distinction between store formats through the fixtures and fittings of the store, which identifies the appearance of a store. The fixtures and fittings depend on various factors, such as the size of the store and the products that are stocked in the store. The stores in the data provided can be divided into 8 store formats, each with a unique fitting code. The number of stores in ascending order for each store format category is listed in Table 4.4.

Store format	Number of stores	Percentage
A	10	0.83%
B	62	5.13%
C	87	7.21%
D	94	7.79%
E	132	10.94%
F	201	16.65%
G	287	23.78%
H	334	27.67%
	1 207	100%

TABLE 4.4: *The store format categories of the considered stores of the Retailer.*

4.1.3 The dataset

The dataset contains the following relevant data fields of information: **region name**, **branch name** and **branch number**, **subclass name** and **subclass number**, **style code**, **style season code**, **last day of the week (LDOW)**, **opening stock quantity** and **BE opening stock sell amount**, **inflow quantity** and **BE inflow sell amount**, **available stock quantity** and **BE available stock sell amount**, **sale quantity** which is made up of **regular sales** and

promotional sales value, BE sale sell amount, BE regular sale sell amount, BE promotional sale sell amount, closing stock quantity and BE closing stock sell amount, stores sales plan value, service level value and the fixture fitting code.

The region specifies the broad location of a store. A store is identified by its store number. Each subclass also has a unique code to identify the product type, known as the subclass number, and to identify the season that that particular style of subclass belongs, known as the style season code.

Sales information is summarised per week, where the last day of the week (LDOW) is specified, as well as the relevant level and movement of stock within that week for a particular style of a subclass. The sales data consists of the opening and closing stock in the store in a given week. The closing stock of one week will become the opening stock for the following week. The difference between opening and closing stock is determined by the amount of inflow of stock which, when combined with the opening stock, specifies the stock available, and also any sale of items. The sales quantity is the net total of regular and promotional sales made in a week. Regular sales represents the number of items sold at full price, and promotional sales shows the number of items sold at a discounted price.

The BE (base exclusive) sell amounts of the relevant inventory levels is the value of the units at that point of the season. The value per item does not remain constant during the period, as some of those products may no longer be sold at full price. There is also the case where different styles within a subclass are not valued at the same prices, and there is no differentiation of which style within a subclass was sold from the data.

Branch No.	Subclass No.	Style season code	LDOW	OpenStock	Inflow	AvailStock	Sales	RegSales	PromSales	CloseStock
H8793	5875	W17	28JAN2017	0	36	36	2	2	0	34
H8793	5875	W17	04FEB2017	34	30	64	4	4	0	60
H8793	5875	W17	11FEB2017	60	0	60	4	4	0	56
H8793	5875	W17	18FEB2017	56	18	74	2	2	0	72

TABLE 4.5: An example of four week's data entries for store H8793 of season W17.

Table 4.5 is an excerpt of the data entries of an arbitrary store to give a better understanding of the data fields given. This example shows the activity of the store for a single subclass of a particular season during the four weeks from 28 January to 18 February 2017. The first entry shows an inflow of 36 items of this particular product in the week ending 28 January 2017 and a sale of 2 units at regular price. The flow of inventory during this week is reflected as such in the closing stock column. In the week ending 4 February 2017, the opening inventory of 34 units with an additional inflow of 30 units makes the combined available stock equal to 64 units. The opening stock of a week is equal to the closing stock of the previous week. The complete data is similar yet more extensive than the four-week extract shown in Table 4.5, as the data is determined over multiple subclasses and seasons for many weeks.

4.2 Data validation

A thorough inspection of the data is conducted to ensure that the effect of data fields on one another is accounted for. The integrity of the data are validated to make conclusions and assumptions that will yield more accurate results. This process is important to ensure that the results from the DEA models is as accurate as possible. The data validation and data analysis processes were executed and completed in SAS.

4.2.1 Data entries

The data need to be checked for any inconsistencies in the data fields. There is no incomplete or empty data entries that have been identified in the data. The completeness of the captured data can thus be assumed for further use and investigation.

4.2.2 Duplicates of data entries

Duplicates of entries exist in the data. Duplicates are identified by comparing each unique data field of every entry, and eliminating the duplicate. The unique identifiers are **branch number**, the **subclass number**, its **season style code** and the **LDOW**. The data identified and removed 62 duplicate entries from the dataset of 2 241 521 data entries.

4.2.3 Negative entries

Negative entries exist in the dataset. Negative entries in the inflow quantity data field can indicate an inter-branch transfer (IBT). Negative entries in the sales quantity data fields, which comprises of the **regular sales** and **promotional sales** data fields, represent returns by customers. Negative entries in the **opening stock** and **closing stock** data fields are an effect of the negative entries that occur in the **inflow quantity** and **sales quantity** data fields, or data that was not correctly captured into the database because of double processing of items when bar codes for similarly priced items are missing. Regular stock-takes will correct these negative entries. The negative entries are considered in the dataset, as they represent real sales transactions.

4.2.4 Disclosure of weeks

Data entries are recorded at the end of each week, which is Saturday for all stores. Data are recorded consecutively and the maximum number of entries for a particular season's subclass that data may be disclosed for is 26 weeks, which is the number of weeks from 28 January to 22 July 2017. There is a store in the data that has 40 entries. Upon further investigation, it was determined that there were two transactions entries for transactions occurring in the same week. It is not clear whether it is an error from the recording of data, or if the data belongs to transactions of another subclass of products, so the entries from this store were omitted from the DEA models. Entries for a store that are less than 26 weeks belong to styles of subclasses that were released later in the season or because there was no sales activity after a certain date due to stock sold out or stock that are no longer being sold.

4.2.5 Uniqueness of store names

The data entries are specified by store name and number. It is important to note that the store names are not unique while store numbers are, since stores are named after the town they are situated in. This implies that multiple stores in a town can have the same branch name or a derivative thereof. Stores are thus referred to by store number instead of by name to ensure the unique identification and investigation of stores.

4.2.6 Flow of inventory

For any given week, the **opening stock** and the **inflow quantity** data fields represents the total inventory that a store has on hand. The **sales** data field represents an outflow of stock during a week, and the **closing stock** data field is the number of products available in store on the last day of the week.

An investigation on whether the values in the data fields balance out was conducted. In some instances, there are stores where there is no inflow or sale during a week, but there is a movement of stock present in the opening and closing stock. This movement between opening and closing inventory can result from theft, damage to goods or stock-take adjustments. This relationship between the data fields can thus be checked and balanced by the equation,

$$\text{OpenStock} + \text{Inflow} - \text{Sales} - \text{Other} = \text{CloseStock}. \quad (4.1)$$

Equation (4.1) corrects the flow of stock, where any change in the stock that is not explicitly identified as inflow or sales is disclosed as **Other**. The variable, **Other**, accounts for exceptions and ensures the data remains credible.

4.3 Data analysis

This section offers a detailed description of each of the relevant data fields. These data fields are relevant to the DEA models and have been utilised in determining the inputs and outputs in the models.

4.3.1 Base exclusive values

The base exclusive (BE) value of stock is the number of units multiplied with the price per unit. There are also other costs that play a role in the pricing of a unit of product. There are markdowns and mark-ups that are considered, which explains the reason why the cost per unit is not constant throughout the flow of inventory. This means that equation (4.1) will not apply as directly in the circumstance where the value of the stock is concerned.

The BE values give an indication of the monetary value of the products. The BE value for products sold is useful for this study to give an approximation of turnover. The **BE sale sell amount** data field is used, and takes into account goods sold at regular price and at promotional price.

4.3.2 Opening and closing stock

The **opening quantity** data field represents the number of items in stock at the beginning of a week. For stores that trade on Sundays, the beginning of the week starts on Sunday. The remaining stores' weeks start on Mondays, unless new stores open on other days, or the products are received from the DC during the week. The opening quantity value for fashion items at the beginning of the season will always be zero, because style items are not carried over to different seasons. The opening quantity value at the start of one week is equivalent to the closing stock quantity of the previous week.

The **closing quantity** data field is the number of items in stock on Saturdays, the last day of the week for all stores. The closing quantity value for one week will be the opening quantity value for the next week, and if the closing quantity is zero, then no stock is available for sale of that particular style and size. A value of zero in the **closing quantity** data field during the season indicates a stock-out of that product.

A negative value in the **closing quantity** data field means that there has been an error in the system, since a store cannot sell stock that it does not have. The negativity that affects closing inventory also impacts the opening inventory for the next week.

A positive value in the **closing quantity** data field means that there is stock on hand. A store that overstocks or under-stocks items of a certain style and size raises concern for the stability of that particular store. The Retailer typically aims for $\pm 10 - 20\%$ stock remaining at the end of a season to use for later styles. Ideally, a store would aim to have little stock on hand by the end of the season, but this should not happen too soon in the season, since it can lead to lost sales.

4.3.3 Inflow quantity

The inflow size profile represents the profile determined during the planning phase of the Retailer's supply chain process for a given season. Table 4.6 shows the number of subclasses within each season, as well as the total number of inflow for each season.

	W17	W16	00
Number of subclasses	44	39	12
Total number of units inflow	4 043 287	54 126	1 112 531

TABLE 4.6: *The number of subclasses and the total number of units inflow per season.*

The **inflow quantity** data field in the datasets contains negative values, which suggests that stock was sent from one store to another store. Although there is no indication of which stores received an IBT from the datasets available, the negative entries in the **inflow** data field indicate where an IBT originates.

Investigation into IBTs is outside the scope of this thesis, but the negative entries will be considered. The assumption is that a negative value in inflows will result in a positive inflow at another store (or multiple stores) at some time during the season, and will not be regarded as a loss of stock. This will also give a more accurate representation of stock when comparing the relationship of inflows with sales regarding the availability of stock.

4.3.4 Sales

The **sales quantity** data field represents the sales of items at normal price and sales made at a discounted price. The differentiation is made in the **regular sales quantity** and **promotional sales quantity** data fields. Regular sales are products sold at full price, and promotional sales are sales sold at a discounted price to customers and to employees.

Items that are returned to a store appear as a negative value in the **sales** data fields. The assumption is made that products are returned to the same store from which it was initially purchased. This negativity will be taken into account when determining the sales profile, since a return in the data implies that an item was, in effect, never sold. When a return is recorded, the quantity of stock increases, which implies that more products are available for sale.

There cannot be negative opening or closing stock for a store because one cannot sell more items than what is available in stock. There are seldom cases where a bar code of an item is not attached to the product, so another product of the same price will be scanned but will remain in the store. This process does not affect the customer, but according to the database, there is a record of a sale that effects the quantities of another product. The result is that the product without a bar code will have less stock in-store than what is reflected on the database, and the product used to price the other without a bar code has more stock on the shelves than is reflected on the database.

Regular sales form the majority of the sales. Table 4.7 shows the percentage of sales within a season that result from sales at regular price and from sales at a discounted price. The percentage of regular sales is 83.23% in W17, 44.23% in W16 and 92.84% for replenishment products.

	W17	W16	00
% Regular sales	83.23%	44.23%	92.84%
% Promotional sales	16.77%	55.77%	7.16%
Total number of sales	2 993 971	402 622	986 912

TABLE 4.7: *The percentage total regular sales and total promotional sales per season.*

The total regular sales of products from new seasons is higher when compared to older seasons' stock, which is more likely to sell at promotional price. This is typical of the life cycle of fashion items, which is given in Figure 4.2. The freshness of subclasses, particularly with fashion items, affects the price that the Retailer will demand for that product. Replenishment items are sold throughout the year, so there is no finite life cycle to the product, which is represented in Figure 4.3. The life cycles of fashion and replenishment are consistent with the description given in § 1.3.

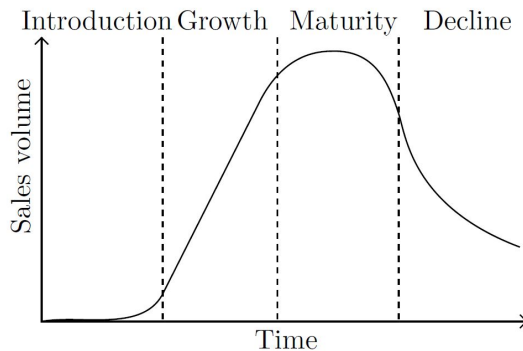


FIGURE 4.2: *The life cycle of a fashion product expressed through the sales over time.*

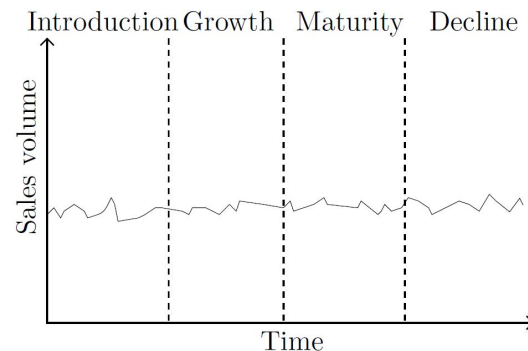


FIGURE 4.3: *The life cycle of a replenishment product expressed through the sales over time.*

4.3.5 Style code

The style code represents the number of styles that are available at that point in time in a store for a particular subclass. Whenever there is an inflow of new stock or when products of a certain style sells out, then the change in the number of styles available will be reflected in the **style code** data field. Styles are not sent to stores simultaneously, but tend to follow one another in waves so as not to overstock stores and to allow for a gradual release of products over an entire season. The number of styles also gives an indication of the width of available options to a store.

4.3.6 Stores sales plan value

The store sales plan is the forecast for a store and subclass per week. This is forecasted by the Retailer and it is what they expected sales to be for any given week, and is usually done 6 months before the season begins. The forecasts are determined by past performance, and provides stores with a target of their sales. These forecasts are made for subclasses that sell per season and not annually. This is used to make forecasts for future subclasses, but based on seasonal subclasses and not replenishment products.

4.3.7 Service level value

The service level value, also known as the availability level, reflects whether the store had enough stock to reach its expected sales, or its store sales plan value. If there is less stock available than what the Retailer forecasted sales to be, then the service level value will be less than the store sales plan value. If there is the right amount of stock available (or more) of a product within a subclass, then the service level value will be equal to the store sales plan value. This service level value depends on many factors, such as different sizes of products and the styles.

4.3.8 Last day of the week (LDOW)

The last day of the week (LDOW), which often falls on a Saturday, is used to count the number of weeks that a subclass has been active for throughout a season. Since the data only record 26 weeks' worth of information, the maximum number of weeks for which a subclass for a particular store (over all styles) is 26 weeks. The number of weeks in the data set can be an indication of the performance of a subclass: a subclass that is sold for a longer period of time has a good chance of many sales relative to a subclass of products that are limited.

CHAPTER 5

DEA components

Contents

5.1	Grouping criteria	54
5.2	Outputs	54
5.2.1	<i>Rate of sales</i>	55
5.2.2	<i>Turnover</i>	55
5.2.3	<i>Store service level</i>	56
5.3	Inputs	56
5.3.1	<i>Width</i>	56
5.3.2	<i>Inflow quantity</i>	57
5.3.3	<i>Full price percentage</i>	57
5.4	Assumptions on the models	57
5.5	The DEA models	58

Allocative efficiency relates to the optimal mix of inputs to produce the relevant outputs [82]. The choice of inputs and outputs used in the DEA models will affect the result obtained based on information available, as inefficiencies can occur depending on which mix of outputs and inputs are used in the models [82]. It is thus essential to select inputs that management can modify easily, and outputs that act as attainable goals for inefficient DMUs to achieve.

Variables that are considered as desirable are often regarded as outputs, while variables that appear undesirable are considered as inputs. This can be subjective depending on the perspective of the retailer or management, as different variables can be interpreted differently for various retailers. Therefore, it is important to select input and output variables in a way that can accurately measure the performance of a DMU while simultaneously managing cost effectiveness [59].

Weights are added to inputs and outputs to make an efficiency score sensitive to the mix of inputs and outputs [82]. This means that in a DEA model with multiple inputs and/or outputs, the most suitable weights are chosen to try and make each DMU as efficient as possible, relative to all other DMUs. This study will consider the inputs and outputs of the Retailer, and apply these components to the DEA model at store-level and at subclass-level.

A Delphi approach was applied when determining variables for DEA, which involved a series of three meetings with executive members of the Retailer, a separate meeting with two planning managers and correspondence with the Retailer's system analyst. The Retailer provided a list of potential input and output measures from the available data which they use when analysing data. The input and output variables identified for the DEA models is limited by the data made available by the Retailer. All input variables, output variables and calculation groups are worth investigating and are relevant for the Retailer [87].

Chapter 5 contains an outline of the grouping of DMUs considered in this thesis in § 5.1. This is followed by brief descriptions of the outputs and inputs considered for the DEA models in § 5.2 and § 5.3, respectively. The chapter concludes with the model specifications used to obtain the DEA results, given in § 5.5.

5.1 Grouping criteria

The DEA models can be run for all DMUs available. An increase in the number of considered DMUs for every calculation group will allow for a positive effect on the discriminatory power and give a more accurate result for the efficiency score of all the DMUs. This study will focus on the results from the DEA model when constant returns to scale and variable returns to scale are considered, and the results of the performance of DMUs in different regions and within subclasses. It is thus more beneficial to a subset of the data to investigate and compare the results from this subset.

The data are divided into regions. There is research and evidence that suggests that geographic concentration leads to innovation and competitive success [68]. Groups of similar and related units within the same geographic area share common markets and technologies [63]. The Retailer clusters their stores into different regions.

The Retailer also categorises products into subclasses. Subclasses are then further split into styles of different seasons, whether it be replenishment products (denoted as style season code 0), or a summer or winter fashion product and the respective year that style was implemented (denoted with SXX or WXX for a summer product of the year 20XX or a winter product of the year 20XX, respectively). The stores and subclasses that will be considered are from products with the style season codes W17 and W16. These are the most recent seasons for fashion products, with season W17 being the most recent season. Replenishment products, which have a steady life cycle, will also be considered in this study.

Furthermore, the Retailer also distinguishes stores by their store format. The store format is an indication of the fixtures and fittings within stores, which provide information of the store appearance. A particular store format is characterised by the focus points within the store, the consolidated payment system and the coverage of products within the store. Stores with similar store formats are grouped together, and will be considered when grouping store DMUs for DEA. Stores with the same store format are considered to have similar store attributes.

The data consist of 15 regions and 8 store formats, all of which will be considered. There are 64 subclasses represented in the data. The season W17 is present in 37 out of the 64 subclasses, and season W16 is present in 39 of the 64 subclasses. There are 11 subclasses that contain replenishment products. Formal clustering methods may be applied to determine calculation groups for the DEA models. This study found that the better approach in determining the most beneficial clustering for the Retailer would be through Delphi approach by consulting the Retailer directly. The Retailer recommended that in addition to the aforementioned grouping criteria, that the top excelling DMUs in terms of turnover be considered as well.

5.2 Outputs

Outputs have been defined as the product resulting from using inputs within a process. Outputs can be attributed to profits or earnings, but more often than not it can also be the achievement of a certain service level or a measure of performance for a given stakeholder. The value of an

output, depending on the orientation of the DEA model, is maximised to get the most benefit from a given level of input for a DMU. The outputs that were considered but were not available from the data obtained from the Retailer were sales growth, profit growth and gross margin return on investment. The following entities might be considered as outputs.

5.2.1 Rate of sales

The rate of sales (ROS) is the average number of sales that occurred per week for the entire season. This is done for each store over all styles of the same season style code when calculating ROS on store-level. The total number of sales for the season of a subclass were used in the calculation of ROS on a subclass level. ROS is used to measure whether the sales within a certain week of the season was similar to the expected ROS and is used for comparability of sales over all DMUs with varying weeks of sale. The data have 26 weeks of recorded sales information (from 28 January 2017 to 22 July 2017), where some styles were only sold later in the season.

All DMUs with weeks spanning more than 26 weeks were removed from the data. This can be due to erroneous capturing of data. This instance only occurred with one DMU, where two different entries were recorded for the same DMU at the same point in time. Since there is not enough information to reconcile this error, the entry of this DMU has been omitted from the study. This will not have a major effect on the DEA model, since the sample size of DMUs is large enough to satisfy the conditions in § 2.5.

5.2.2 Turnover

Turnover is the total value of sales for a DMU during the period 28 January 2017 to 22 July 2017. The turnover value cannot simply be calculated by multiplying the value of a sale with the number of units sold. This is because a style can differ from another within a subclass, and there is no specification of the particular style being sold in the data. The variance in the individual value of a unit sold may also differ from another unit because of products sold at a marked down price. Despite this notion, there is a strong positive correlation between turnover and the rate of sales and a p -value less than 0.0001 for all the correlation statistics that were tested. The correlation coefficients of the Pearson correlation coefficient, the Kendall rank correlation and the Spearman rank correlation between turnover and the rate of sales is given in Table 5.1.

Correlation statistic	Correlation coefficient
Pearson	0.99513
Kendall	0.95028
Spearman	0.99482

TABLE 5.1: *The correlation coefficients of the relationship between turnover and rate of sales.*

It is recommended that in the case of two variables that are highly correlated, on such variable should be removed to avoid the loss of discriminatory power [24]. Although there is a strong positive correlation between these two variables, Dyson *et al.* [33] state that there is a significant impact on the efficiency impact on the efficiency scores of some service units when a highly correlated variable¹ is omitted from the model. It is assumed that the value as is recorded is accurate and the sum of all values of the units sold is summed together to represent turnover. The Retailer would want to maximise turnover for a favourable result by the end of the season.

¹Highly correlated variables are considered to have a correlation coefficient greater than 0.8 [61].

Turnover for stores as DMUs is calculated over all subclasses within a particular season, whereas when subclasses are the DMUs for DEA, the turnover is determined over the entire subclass within a specific style season code.

5.2.3 Store service level

The store service level is a percentage of how well a DMU (particularly a store) is stocked within a season. In other words, it is the amount of stock available as a fraction of the expected sales of a store. This percentage changes as more stock is sold throughout the season, because the available stock fluctuates as a result. The Retailer has a plan for what it expects sales to be for each service unit. The store service level indicates if there was enough stock available to satisfy the expected sales plan.

A higher value of the store service level means that the sales potential of stock, which is the available stock, is closer to what it is expected to be. The average store service level of all the weeks of a DMU were used as an output on subclass-level. The data do not have values for **service level value** and **store sales plan value** with replenishment product typed subclasses or yearly merchandise typed subclasses. These types of products have established sales patterns because they repeat annually or they are constantly replenished throughout the year, thus there is no sales plan value. This means that DEA for replenishment products will be conducted with one less output than the DEA models for fashion products.

5.3 Inputs

Inputs have been defined as a resource that is used within a process. Conversely to outputs, inputs are often attributed to be costs and expenditure, but can be any resource or commodity that contributes to the production of outputs. The value of inputs are minimised to ensure that there is a saving in these resources, but only if it does not affect the level of output produced by a DMU.

Inputs are often within a stakeholder's power to be modified, altered or controlled. The inputs that were considered but were not available from the data obtained from the Retailer were general overheads, such as rent and labour expenses, supplier efficiency, best price leadership information, the market size or population and the density of stores within proximity of one another. Efficiency can be improved by identifying how inputs must be controlled or regulated while still maintaining the decision maker's current output level. Potential inputs are discussed in the following subsections.

5.3.1 Width

The width of products is the number of styles available to a specific DMU. A subclass comprises of multiple styles of products. The size of a subclass can indicate the size of a store, since a larger variety of products within a subclass can give an indication of the popularity of that subclass for that store.

The Retailer is able to change the styles made available at each store, and is thus able to change the width of products of that subclass made available to consumers. The maximum number of styles between 28 January 2017 to 22 July 2017 is used for this calculation to represent the

maximum number of styles that the DMU can accommodate at once within a season of styles in a subclass.

It is possible to consider the width of the store or subclass as a means of clustering or grouping DMUs of the same width together in DEA instead of considering the width as an input. Removing width as an input will have a positive influence on the discriminatory power of the model, but the correlation between the width and the other input variables is weak², which means that the efficiency scores will be impacted even more than if there was a strong correlation, and may not represent the efficiency of DMUs accurately.

5.3.2 Inflow quantity

The inflow quantity is the amount of stock that is received by a DMU each week. A store will receive different styles of a subclass at different times during the season. This is the case for fashion products, whereas replenishment products are received by stores on a replenishment cycle.

The inflow quantity is calculated as the sum of all inflows for a DMU per style season code; thus, all the inflows that a store has received over the time period 28 January 2017 to 22 July 2017. The inflow quantity at store-level and at subclass-level are both calculated in this way.

5.3.3 Full price percentage

The full price percentage is an indication of markdowns for a certain DMU or where stock had been written off. The percentage itself is a fraction of the quantity of stock that was sold at a markdown or at promotional price relative to the total number of sales.

A percentage of 0% means that all of the sales that occurred for that DMU were sold at its full price, and 100% means that no sales were made at full price. This input should be minimised as much as possible by the Retailer, since it is better to sell at full price than at a reduced price.

5.4 Assumptions on the models

The building of DEA models requires important considerations to keep in mind. As a result, some assumptions must be made when considering what the model must achieve. The following assumptions on the DEA models are made:

1. The inputs, outputs and choices of DMUs should be a reflection of the decision maker's interest in these components, in order for the relative efficiency scores to be relevant,
2. The units of measurement for different inputs and outputs need not be the same. This implies that they exhibit unit incongruence, which means that some variables may be monetary and other variables may be physical in nature,
3. In principle, smaller input amounts are preferable and larger output amounts are preferable, and hence the efficiency scores should be a reflection of these principles,
4. The number of DMUs considered in a calculation group must comply with the criteria proposed in literature in § 2.5,

²A weak correlation between variables is when the correlation coefficient is between 0 and ± 0.35 [85].

5. DMUs in the same calculation group are assumed to be operating in similar operating environments, and
6. All inputs and outputs are considered to be discretionary variables, meaning that they can be varied at the discretion and are under the control of management [24].

5.5 The DEA models

The output-orientated DEA model will be used for this study. This ensures that the results obtained from the model are able to be identified and implemented by management. The output-orientated model allows the Retailer to identify inputs that are not being utilised efficiently, and can adjust them and improve productivity and save resources simultaneously. It is thus important to identify DMUs that are underperforming and to identify ways to maximise the output level of a DMU. In this way, a decision maker has control over improving the performance of its decision-making units. An output-oriented model for this particular study is also preferable over an input-oriented model, since an input-oriented model would retrospectively identify what the level of input should have been, given a level of output. An output-oriented model will be better in identifying the DMUs that are producing output at an inefficient standard, and will allow the Retailer to monitor the actual performance of these DMUs.

The DEA models for all criteria were run on an Intel(R) Core(TM) i7-7700 CPU @ 3.60GHz with 8.00GB installed memory (RAM), a 64-bit operating system and x64-based processor. The computer uses Windows 10 Enterprise 2016 and 64-bit SAS Enterprise Guide Version 7.11 was used to process the data and extract the necessary results. The DEA model was adapted from the SAS Institute Inc. User's Guide entitled *Efficiency Analysis: How to Use Data Envelopment Analysis to Compare Efficiencies of Garages* and by Lancheros *et al.* [10, 52, 77].

CHAPTER 6

Results

Contents

6.1	DEA for store DMUs	59
6.1.1	<i>Performance of store formats</i>	60
6.1.2	<i>Stores with products of season W17</i>	63
6.1.3	<i>Stores with products of season W16</i>	66
6.1.4	<i>Stores with replenishment products</i>	68
6.2	DEA for subclass DMUs	70
6.2.1	<i>Subclasses of season W17</i>	71
6.2.2	<i>Subclasses of season W16</i>	72
6.2.3	<i>Subclasses of replenishment products</i>	73
6.3	Summary of results	73

The aim of DEA is to determine efficiency relative to other DMUs. It is a powerful benchmarking tool that determines how well an individual DMU does when compared to every other considered DMU. While DEA does give an efficiency score for the performance of a DMU based on its use of inputs and its level of output, the scores do not give rankings of the DMUs. The efficiency score is there to give an indication of how similar or dissimilar a DMU is, compared to efficiently performing DMUs.

The objective of this chapter is to identify units that are performing well and service units that have the ability to improve. DEA results were obtained for different regions, store formats and products of specific seasons to allow for more comparable observations. The efficiency scores under CRS and VRS are also obtained to show the difference that both returns to scale produce.

Chapter 6 begins with the results of DEA when considering stores as DMUs. The results are subject to the calculation groups as described in § 5.1. The results contains information regarding the performance of the calculation groups of stores in § 6.1. The results of DEA when the DMUs for the DEA models are subclasses is given in § 6.2. The chapter concludes with a summary of the results in § 6.3.

6.1 DEA for store DMUs

The DEA models were run on the stores of the Retailer to determine how well the stores set as DMUs perform relative to other stores in terms of efficiency. DEA will provide the efficiency scores of all products in stores from the same sales season. The data contain sales information from 1 207 stores, of which all stores sell different styles because of historical sales information, geographic location, etc. Thus, each season will be considered separately.

6.1.1 Performance of store formats

The efficiency of stores with the same store formats is considered for comparability and sensitivity. The efficiency scores for all products over all seasons within the baby boys' outerwear department is determined. The inputs and outputs for this grouping includes the total sales data from all seasons and all subclasses, and not just the products of particular seasons or subclasses, to determine the overall efficiency scores of stores.

The average efficiency score of all 1 207 stores, considering all of the data and all stores set as DMUs, is $\bar{\theta}_{CRS} = 0.82524$ with 27 of the 1 207 stores being identified as efficient under CRS, and $\bar{\theta}_{VRS} = 0.89101$ with 57 stores identified as efficient under VRS. The standard deviations of efficiency scores under CRS and VRS are $\sigma_{CRS} = 0.10174$ and $\sigma_{VRS} = 0.06756$, respectively. The standard deviation provides insight into the spread of the efficiency scores. A standard deviation that is close to zero means that the data points (in this case, the efficiency scores), tend to be close to the mean of the set, while a high standard deviation indicates that the efficiency scores are spread out over a wider range of values.

The average performance of stores when grouped by store format, as well as the number of stores that performed efficiently within those groups according to DEA under CRS and VRS respectively, are shown in Table 6.1 in ascending order of the number of stores considered within each of the store format calculation groups. Store formats, as mentioned in § 5.1, are considered to group stores with similar store attributes, and will hence provide insight into the performance of “types” of stores, or similar stores. The efficiency scores for stores of the “A” store format could not be determined since the number of DMUs does not satisfy the required number of DMUs for accurate benchmarking, as stated in § 2.5. The efficiency scores for individual stores within each store format is shown in Tables A.1–A.13.

Store format code	Number of stores	$\theta_{CRS} = 1$	$\bar{\theta}_{CRS}$	$\theta_{VRS} = 1$	$\bar{\theta}_{VRS}$
A	10	N/A	N/A	N/A	N/A
B	62	12	0.918 21	20	0.956 15
C	87	13	0.874 34	22	0.921 28
D	94	20	0.938 22	31	0.963 47
E	132	21	0.925 27	39	0.951 36
F	201	22	0.889 52	35	0.927 20
G	287	32	0.919 41	49	0.940 40
H	334	30	0.901 74	41	0.918 34

TABLE 6.1: Average efficiency scores of store formats under CRS and VRS.

For interest's sake, a multiple linear regression model is applied to the data of all the stores with the average efficiency score of the sample of DMUs as the dependent variable and all the values of the input variables as independent variables. The regression equations are given by

$$\hat{\theta}_{CRS} = 0.96144 + 0.00018130 x_1 - 0.00515000 x_2 - 0.40900 x_3, \text{ and} \quad (6.1)$$

$$\hat{\theta}_{VRS} = 0.91332 + 0.00009811 x_1 + 0.00061940 x_2 - 0.20626 x_3, \quad (6.2)$$

where x_1 is the value of inflow quantity input, x_2 is the value of the width input and x_3 is the value of the full price percentage input. The regression equations (6.1) and (6.2) may be interpreted as calculating the expected efficiency score for a DMU while considering multiple input variables at once, and the effect that a unit increase in one of the inputs will have on the efficiency score, given that all other variables remain constant. The coefficient associated with the full price percentage input, x_3 , is much higher than the coefficients of the other input variables in both regression equations. This is because variable x_3 is a percentage, and hence

a unit increase is an increase of 0.01, or 1%. The accompanying statistics of the regression equations (6.1) and (6.2) is given in Tables 6.2 and 6.3, respectively.

Variable	<i>t</i> value	<i>p</i> value
Intercept	65.28	< 0.0001
x_1	8.92	< 0.0001
x_2	-6.60	< 0.0001
x_3	-13.07	< 0.0001

TABLE 6.2: Regression statistics of inputs under CRS.

Variable	<i>t</i> value	<i>p</i> value
Intercept	87.82	< 0.0001
x_1	6.83	< 0.0001
x_2	1.12	0.2614
x_3	-9.34	< 0.0001

TABLE 6.3: Regression statistics of inputs under VRS.

By the *t*-values and the *p* values of both tables, the null hypothesis is rejected and it is shown that there is statistical significance between the input variables and the efficiency score, except for the width input under VRS. Similarly, multiple linear regression is also applied to the data with the average efficiency score of the sample of DMUs as an independent variable and all the values of the output variables as independent variables. The regression equations are given by

$$\hat{\theta}_{CRS} = 0.49192 + 0.00057149 y_1 - 0.00000041 y_2 + 0.66669 y_3, \text{ and} \quad (6.3)$$

$$\hat{\theta}_{VRS} = 0.68484 + 0.00003318 y_1 + 0.00000012 y_2 + 0.42023 y_3, \quad (6.4)$$

where y_1 is the value of the rate of sales output, y_2 is the value of the turnover output and y_3 is the value of the store service level output. The regression equations (6.3) and (6.4) may be interpreted as calculating the expected efficiency score for a DMU while considering multiple output variables at once, and the effect that a unit increase in one of output will have on the efficiency score, given that all other variables remain constant. Once again, the coefficient associated with the store service level output, y_3 , is much higher than the coefficients of the other output variables, because variable y_3 is a percentage. Hence, a unit increase is an increase of 0.01, or 1%. The accompanying statistics of the regression equations (6.3) and (6.4) is given in Tables 6.4 and 6.5, respectively.

Variable	<i>t</i> value	<i>p</i> value
Intercept	29.93	< 0.0001
y_1	2.98	0.0030
y_2	-1.85	0.0653
y_3	17.25	< 0.0001

TABLE 6.4: Regression statistics of outputs under CRS.

Variable	<i>t</i> value	<i>p</i> value
Intercept	60.50	< 0.0001
y_1	0.25	0.8020
y_2	0.78	0.4377
y_3	15.79	< 0.0001

TABLE 6.5: Regression statistics of outputs under VRS.

Upon investigating the statistics in Tables 6.3 and 6.4, it is noted that the rate of sales output, y_1 , and the turnover output, y_2 , are not significant to the regression models. This is because, as is discussed in § 5.2.2, there is a strong correlation between these two variables, and the model would be significant if one of these correlated variables were removed from the model. The regression equations when the rate of sales output is removed are given by

$$\hat{\theta}_{CRS} = 0.45763 + 0.00000022 y_2 + 0.72378 y_3, \text{ and} \quad (6.5)$$

$$\hat{\theta}_{VRS} = 0.65387 + 0.00000014 y_2 + 0.46152 y_3, \quad (6.6)$$

where y_2 is the value of the turnover output and y_3 is the value of the store service level output. The regression statistics after removing one of the highly correlated variables, ROS, under CRS and VRS are given in Table 6.6 and 6.7, respectively.

Variable	t value	p value
Intercept	27.49	< 0.0001
y_2	9.18	< 0.0001
y_3	18.69	< 0.0001

TABLE 6.6: *Regression statistics of outputs under CRS without ROS.*

Variable	t value	p value
Intercept	56.25	< 0.0001
y_2	8.28	< 0.0001
y_3	17.07	< 0.0001

TABLE 6.7: *Regression statistics of outputs under VRS without ROS.*

The t -values and the p values in Tables 6.6 and 6.7 after a correlated variable is removed indicates that the null hypothesis is rejected therefore there is statistical significance between the output variables and the efficiency score.

Regression model	R^2	p value
Inputs under CRS	0.2840	< 0.0001
Inputs under VRS	0.1905	< 0.0001
Outputs under CRS	0.3590	< 0.0001
Outputs under VRS	0.3108	< 0.0001

TABLE 6.8: *Regression statistics for the regression models.*

The values of the statistical tests on the accuracy of the regression models are given in Table 6.8. The values of R^2 for all the regression models are below 0.4, and an R^2 value close to one is preferable. The value of R^2 means that the variation in the efficiency scores is not explained by the independent variables. While the regression models prove that there is statistical significance of the variables in the models, it is expected that the R^2 value is low, because the regression models can only consider multiple inputs or multiple outputs at once, and therefore the R^2 value is only considering the effect of multiple inputs at a time, or multiple outputs at a time, and not the effect of both inputs and outputs at once.

Store format code	θ_{CRS}	Average inputs			Average outputs		
		Inflow	Width	Full price %	ROS	Turnover	SSL
D	0.938 22	235	16	0.207 56	205	165 580	0.472 45
E	0.925 27	208	15	0.216 80	171	134 738	0.465 19
G	0.919 41	195	15	0.214 05	168	133 427	0.464 29
B	0.918 21	193	14	0.224 85	160	126 769	0.450 77
H	0.901 74	177	14	0.225 41	148	117 505	0.452 75
F	0.889 52	107	13	0.268 93	89	68 789	0.421 38
C	0.874 34	94	12	0.296 00	76	58 238	0.418 90

TABLE 6.9: *The average inputs and average outputs of each store format.*

Table 6.9 contains a list of the averages for all the inputs and outputs of the DMUs from each store format considered in this thesis. The store formats are listed in descending order of average efficiency scores under CRS. A relationship between each input and each output may be seen when each store format is considered. For instance, when considering each store format in descending order of its efficiency scores, there is also a descending trend in the average inflow quantity and the average width, and an increasing trend in the full price percentage. Similarly when considering the outputs, there is an decreasing trend in the rate of sales, the total turnover and the store service level when considering the descending efficiency scores of the different store formats. This can be attributed to the fact that, excluding the full price percentage input, a higher value of each input and output is preferable to a lower value. It is preferable to have a lower full price percentage value, which will indicate that more sales were made at full price.

6.1.2 Stores with products of season W17

There are 1201 stores that received and sold products from season W17. DEA identified 11 DMUs that performed efficiently from the calculation group of 1201 DMUs for the season W17 under CRS, which is 0.9159% of the DMUs that are efficient. The average efficiency score for DMUs under CRS is $\bar{\theta}_{CRS} = 0.73833$, with a standard deviation of $\sigma_{CRS} = 0.10459$. DEA identified 14 efficient DMUs under VRS, which is 1.1657% of the sample, and the stores have an average efficiency score of $\bar{\theta}_{VRS} = 0.81355$ and standard deviation $\sigma_{VRS} = 0.08645$.

A concern with DEA models is often that if all DMUs are able to adopt their most favourable weights in order to appear efficient, then all DMUs will become efficient [34, 86]. A smaller sample of DMUs relative to the number of input and output variables makes it easier for a DMU to be assigned a high efficiency by the DEA model [48]. This is because the greater the number of DMUs to benchmark against, the more constraints are added to the LP for solving the efficiency score for a specific DMU. This means that there is more discriminatory power in the DEA models, and more DMUs for benchmarking that are considered. If the efficiency score for the majority of DMUs is below 0.9, then there is considered to be a fair degree of discrimination [86].

Top and bottom 20 turnover stores

The efficiency scores of the 20 stores from all regions with the highest turnover from sales of products from season W17 is shown in Table 6.10. The stores satisfies the criteria for the number of DMUs suggested by literature in § 2.5. This smaller sample of DMUs will allow each DMU to be assigned a higher efficiency score by the DEA models [48, 90].

Store ID	Turnover (in R)	θ_{CRS}	θ_{VRS}
W170338	1 644 430.69	1.000 00	1.000 00
W178102	1 269 113.18	1.000 00	1.000 00
W176378	629 268.03	0.872 42	0.949 19
W178727	454 452.34	0.972 19	0.972 21
W176627	444 051.80	1.000 00	1.000 00
W178227	428 009.23	0.958 40	0.970 20
W176272	422 824.02	0.941 43	0.987 59
W178430	415 375.53	1.000 00	1.000 00
W176340	415 221.00	0.993 80	1.000 00
W170982	410 332.75	0.933 74	0.943 18
W178063	398 369.35	0.996 14	1.000 00
W178660	392 848.75	1.000 00	1.000 00
W170551	380 789.61	1.000 00	1.000 00
W176247	373 112.61	0.972 28	0.992 73
W178272	369 832.71	1.000 00	1.000 00
W170211	366 411.10	0.966 48	0.974 56
W170607	343 772.82	1.000 00	1.000 00
W170428	337 056.21	0.998 32	1.000 00
W178499	335 230.66	1.000 00	1.000 00
W178793	330 010.07	1.000 00	1.000 00

TABLE 6.10: The efficiency scores of the top 20 turnover stores of products from season W17 under CRS and VRS.

The DEA models identified 10 efficient DMUs from the sample of the 20 stores with the highest turnovers under CRS with an average efficiency score of $\bar{\theta}_{CRS} = 0.98026$, and standard deviation $\sigma_{CRS} = 0.03305$. The models also identified 13 efficient DMUs when VRS is considered, and the

Store ID	Turnover (in R)	θ_{CRS}	θ_{VRS}
W170955	12 151.80	1.000 00	1.000 00
W176656	11 618.20	0.922 65	0.996 16
W170236	11 592.35	0.888 07	1.000 00
W170417	10 780.74	0.859 43	0.933 52
W170290	10 685.94	1.000 00	1.000 00
W178056	10 338.50	0.969 62	0.989 41
W176516	10 150.52	0.919 78	0.937 49
W176638	9 879.20	0.963 90	0.969 83
W170228	9 796.39	0.811 37	0.860 72
W178697	9 779.81	1.000 00	1.000 00
W176710	9 542.87	1.000 00	1.000 00
W176670	9 422.41	0.946 06	1.000 00
W170594	9 274.47	0.774 33	0.853 13
W178696	9 150.97	0.872 15	0.938 99
W176426	9 146.17	0.836 65	0.841 30
W176473	8 818.62	0.898 85	0.913 33
W178059	7 374.42	0.697 69	0.788 22
W178042	7 224.70	0.631 34	0.675 90
W170598	6 528.14	1.000 00	1.000 00
W176568	3 377.26	0.760 76	1.000 00

TABLE 6.11: *The efficiency scores of the bottom 20 turnover stores of products from season W17 under CRS and VRS.*

average efficiency score under VRS is $\bar{\theta}_{VRS} = 0.98948$ and standard deviation $\sigma_{VRS} = 0.01794$. That means that under both RTS, the DEA models identified 50% or more efficient DMUs with a smaller calculation group, and efficiency scores for all stores are greater than 0.9.

The efficiency scores of stores from all regions with the lowest turnover from sales of products from season W17 is shown in Table 6.11. The DEA models identified 5 efficient DMUs from the calculation group of the 20 stores with the lowest turnovers under CRS with an average efficiency score of $\bar{\theta}_{CRS} = 0.88763$ with a standard deviation of $\sigma_{CRS} = 0.10800$, and 8 efficient DMUs when VRS is considered, with an average efficiency score of $\bar{\theta}_{VRS} = 0.93490$ and standard deviation $\sigma_{VRS} = 0.08941$.

It is worth noting that turnover does not accurately reflect the performance of a DMU when there are multiple inputs and outputs considered. Tables 6.10 and 6.11 show that regardless of the turnover, the efficiency of a DMU is dependent on the other DMUs in the sample and the inputs and outputs used in the DEA models. It is therefore important to considered multiple factors, financial or otherwise, when measuring performance of service units.

Efficiency reference set

The efficiency reference set (ERS), or peer group, of an inefficient DMU is the subgroup of efficient DMUs that provide a “target” for the inefficient DMU to strive towards in order to become efficient. The ERS identifies the efficient DMUs against which each inefficient DMU is found to be most directly inefficient. The ERS of the inefficient DMUs from considering the results of DEA from the 20 stores with the lowest turnover from season W17 is given in Table 6.12.

The inefficient DMUs in Table 6.12 is ordered by descending efficiency scores, and the scores under VRS are considered. The efficient DMUs that are determined by DEA are listed as columns in the table, and the ERS of each inefficient DMU is listed in every row. The values

Inefficient DMU	θ_{VRS}	Efficient DMUs							
		$\lambda_{W170236}$	$\lambda_{W170290}$	$\lambda_{W170598}$	$\lambda_{W170955}$	$\lambda_{W176568}$	$\lambda_{W176670}$	$\lambda_{W176710}$	$\lambda_{W178697}$
W176656	0.9962	0.2413	0.2413	-	0.5173	-	-	-	-
W178056	0.9894	-	0.4683	0.0869	0.2224	-	-	-	0.2224
W176638	0.9698	-	0.7853	0.1261	0.0443	-	-	-	0.0443
W178696	0.9390	-	-	0.1367	0.2068	-	0.2248	-	0.4317
W176516	0.9375	-	0.4948	-	0.2526	-	-	-	0.2526
W170417	0.9335	-	0.0149	0.1034	0.8817	-	-	-	-
W176473	0.9133	-	0.3930	0.2705	0.1682	-	-	-	0.1682
W170228	0.8607	0.1750	0.0750	0.1000	0.6500	-	-	-	-
W170594	0.8531	0.0560	-	0.0560	0.5457	-	0.3423	-	-
W176426	0.8413	-	0.3371	-	0.3314	-	-	-	0.3314
W178059	0.7882	-	-	0.3955	0.3950	-	0.2096	-	-
W178042	0.6759	0.4633	0.4364	0.1003	-	-	-	-	-

TABLE 6.12: The efficiency reference set of the bottom 20 turnover stores from season W17.

that correspond to an inefficient DMU j and an efficient DMU represents the relative weight, λ_j , assigned to each member of the ERS to calculate the efficiency score, θ . These weights, which are determined by solving the enveloping model or dual of the DEA model, can also be seen as the mix of weights for each efficient DMU that is needed to make a composite DMU that produces more output with the same input as the inefficient DMU, or save on input consumption while still producing the same output level [82].

The ERS is determined under VRS in an output-oriented model. This means that the values of λ_j should be applied to the outputs of the efficient DMUs to determine a virtual or composite DMU that utilises the same level of input as the considered inefficient DMU at a greater level of output. For example, the inefficient DMU store W176656 is directly inefficient to the store W170236, store W170290 and store W170955. Therefore, the given λ_j values are then applied to the outputs of the efficient DMUs, similarly to the example in § 3.4.2 while using equations (3.28) and (3.29), to produce new outputs for store W176656 to use in order to perform efficiently. Similarly, the ERS can be applied, determined and interpreted for all the results of the DEA models.

Regional performance of stores

The efficiency of stores on a regional level is considered for comparability and sensitivity. The 1 201 stores are grouped into regional subsets based on the data from the Retailer. The average performance of stores within each region for products from season W17, as well as the number of stores that performed efficiently according to DEA under CRS and VRS respectively, is shown in Table 6.13. The efficiency scores for each store within each region is shown in Tables B.1–B.15.

The smallest percentage of efficient stores of a region out all the considered regions is in the Langeberg region, with 3.488% of the 86 stores in that region identified as efficient under CRS, and 8.140% of those 86 stores efficient under VRS. The smallest percentage of efficient stores with smaller DMU calculation groups is still a greater percentage than the entire calculation group of 1 201 stores. The average efficiency score over all DMUs with products of season W17 under CRS ($\bar{\theta}_{CRS} = 0.73833$) and VRS ($\bar{\theta}_{VRS} = 0.81355$) is also improved for all regions of smaller calculation groups, with the lowest average efficiency score for a region, again Langeberg, under CRS and VRS being $\bar{\theta}_{CRS} = 0.79518$ and $\bar{\theta}_{VRS} = 0.88181$, respectively.

Region	Number of stores	$\theta_{CRS} = 1$	$\bar{\theta}_{CRS}$	$\theta_{VRS} = 1$	$\bar{\theta}_{VRS}$
Swaziland	20	9	0.957 16	12	0.981 24
Southern Namibia	41	15	0.940 83	15	0.961 40
Northern Namibia	52	9	0.933 09	15	0.949 16
Botswana	68	12	0.933 06	16	0.944 54
Cederberg	85	16	0.911 87	23	0.942 81
Kwena	85	21	0.962 17	33	0.977 23
Emfuleni	86	12	0.886 92	25	0.955 64
Langeberg	86	3	0.795 18	7	0.881 81
North West	88	12	0.909 48	20	0.934 10
Free State	91	12	0.911 64	25	0.938 41
Lesedi	91	13	0.908 77	24	0.937 66
Gauteng	98	18	0.928 88	27	0.960 03
Limpopo	98	13	0.900 16	19	0.939 21
Thekwini	105	21	0.910 46	28	0.940 08
Tugela	107	18	0.934 74	26	0.949 54

TABLE 6.13: Average efficiency scores of stores in regions of season W17 under CRS and VRS.

6.1.3 Stores with products of season W16

There are 1 195 stores that have received and sold products from season W16. DEA identified 11 DMUs that performed efficiently from the sample of 1 195 DMUs under CRS. The average efficiency score for DMUs under CRS is $\bar{\theta}_{CRS} = 0.26856$ and standard deviation $\sigma_{CRS} = 0.15422$. Comparatively, DEA under VRS shows that there are 19 efficient DMUs and stores with products from season W16 have a total average efficiency score of $\bar{\theta}_{VRS} = 0.29110$, with a standard deviation of $\sigma_{VRS} = 0.17148$.

The data for inputs and outputs are based on sales information from the year 2017, so products from season W16 are considered as less fashionable and from an older season. Thus there are less inflows in general expected to be received at stores for these products, and they will often be sold at a discounted price. This explains the lower efficiency scores in season W16 products when compared to products from season W17.

Top and bottom 20 turnover stores

The efficiency scores of stores from all regions with the highest and lowest turnover from sales of products from season W16 is shown in Tables 6.14 and 6.15, respectively. The number of efficient DMUs under CRS and VRS for the 20 stores with the highest turnover for the season W16 is 6 and 11, respectively and the average efficiency scores for each is $\bar{\theta}_{CRS} = 0.80992$ under CRS and $\bar{\theta}_{VRS} = 0.88580$ under VRS. The standard deviations under CRS and VRS are $\sigma_{CRS} = 0.17515$ and $\sigma_{VRS} = 0.15612$, respectively. Similarly, the number of efficient DMUs under CRS and VRS for the lowest turnover stores in W16 is 4 and 12, respectively. The respective average efficiency scores are $\bar{\theta}_{CRS} = 0.59077$ and $\bar{\theta}_{VRS} = 0.88451$ for CRS and VRS, with standard deviations $\sigma_{CRS} = 0.29400$ and $\sigma_{VRS} = 0.24582$, respectively.

This contributes to the notion that when considering DEA models under variable returns, then more DMUs will be identified as efficient, since the efficient frontier takes increasing and decreasing RTS into account as well. The percentage of stores that are efficient also improves for both the highest and lowest turnover sets of DMUs when compared to the percentage of efficient stores for the larger samples.

Store ID	Turnover (in R)	θ_{CRS}	θ_{VRS}
W168102	65 074.92	1.000 00	1.000 00
W160338	58 950.38	1.000 00	1.000 00
W166358	49 857.50	0.750 24	0.852 11
W166378	45 093.39	0.824 74	0.850 22
W168424	42 157.66	0.872 68	0.875 60
W164244	37 900.24	0.584 78	0.614 20
W166701	37 138.77	0.877 04	1.000 00
W168111	33 412.80	1.000 00	1.000 00
W168603	32 856.35	0.420 56	0.542 40
W166456	32 617.41	0.777 26	1.000 00
W166167	31 687.10	0.927 92	1.000 00
W168272	31 226.95	0.580 79	0.687 85
W166606	31 066.64	0.730 62	0.862 01
W168227	30 262.04	0.815 46	1.000 00
W166639	30 160.96	1.000 00	1.000 00
W168591	30 010.69	0.797 05	1.000 00
W160190	27 931.11	1.000 00	1.000 00
W166330	27 794.42	1.000 00	1.000 00
W160607	27 711.39	0.568 57	0.597 75
W166366	27 007.11	0.670 73	0.833 86

TABLE 6.14: The efficiency scores of the top 20 turnover stores of products from season W16 under CRS and VRS.

Store ID	Turnover (in R)	θ_{CRS}	θ_{VRS}
W164509	1 092.65	0.502 49	1.000 00
W160160	1 085.96	0.591 04	1.000 00
W160911	1 083.83	0.321 60	0.995 16
W168263	1 070.74	0.302 99	1.000 00
W166426	1 069.87	0.449 00	1.000 00
W160778	1 061.96	0.404 85	0.975 54
W160597	1 056.24	0.797 67	1.000 00
W166672	968.24	0.833 57	1.000 00
W160520	938.26	0.653 84	1.000 00
W160746	822.52	0.390 03	0.805 83
W166543	781.08	0.335 27	0.831 44
W166568	733.09	0.266 23	0.675 88
W166503	732.20	1.000 00	1.000 00
W168288	705.86	1.000 00	1.000 00
W166638	588.03	1.000 00	1.000 00
W168105	444.58	0.734 02	0.930 16
W166516	409.93	1.000 00	1.000 00
W160167	370.03	0.830 01	1.000 00
W166710	184.05	0.370 36	0.434 51
W165617	13.15	0.032 48	0.041 67

TABLE 6.15: The efficiency scores of the bottom 20 turnover stores of products from season W16 under CRS and VRS.

Regional performance of stores

The average performance of stores within each region for products from season W16 for DEA under CRS and VRS respectively, is shown in Table 6.16.

The efficiency scores for each store within each region is shown in Tables B.16–B.30. The average efficiency score per region, as is the case with products from season W17, are higher in each region

Region	Number of stores	$\theta_{CRS} = 1$	$\bar{\theta}_{CRS}$	$\theta_{VRS} = 1$	$\bar{\theta}_{VRS}$
Swaziland	19	11	0.894 44	14	0.923 57
Southern Namibia	38	6	0.529 24	8	0.645 76
Northern Namibia	49	10	0.636 88	18	0.703 55
Botswana	68	11	0.673 93	22	0.748 17
Cederberg	85	8	0.571 89	11	0.611 46
Emfuleni	86	13	0.687 65	18	0.750 01
Kwena	86	8	0.580 84	10	0.595 12
Langeberg	86	6	0.527 02	12	0.587 85
North West	88	14	0.697 58	20	0.770 95
Free State	91	6	0.571 50	17	0.653 77
Lesedi	91	11	0.651 81	22	0.729 45
Gauteng	98	19	0.576 51	21	0.662 89
Limpopo	98	9	0.532 25	12	0.569 18
Thekwini	105	6	0.361 94	10	0.446 38
Tugela	107	11	0.518 09	13	0.555 27

TABLE 6.16: Average efficiency scores of stores in regions of season W16 under CRS and VRS.

than when compared to the average efficiency of all of the 1 195 stores of $\bar{\theta}_{CRS} = 0.26856$ under CRS and $\bar{\theta}_{VRS} = 0.29110$ under VRS, despite the generally lower efficiency scores of DMUs for stores of season W16 products.

6.1.4 Stores with replenishment products

There are 1 201 stores that have received and sold replenishment products. It is important to note that in the case of replenishment products, the `store service level` output for all replenishment products is zero, and thus only 2 inputs are considered in the DEA models. DEA identified 27 DMUs that performed efficiently from the calculation groups of 1 201 DMUs under CRS. The average efficiency score for DMUs under CRS is $\bar{\theta}_{CRS} = 0.66112$ and standard deviation $\sigma_{CRS} = 0.17206$. Comparatively, DEA under VRS identified 34 efficient DMUs with an average efficiency score of $\bar{\theta}_{VRS} = 0.68576$ and standard deviation $\sigma_{VRS} = 0.16780$.

Top and bottom 20 turnover stores

The efficiency scores of stores from all regions with the highest and lowest turnover from sales of all replenishment products are shown in Tables 6.17 and 6.18, respectively.

The stores satisfies the criteria for the number of DMUs suggested by literature in § 2.5. The DEA models identified 9 efficient DMUs from the sample of the 20 stores with the highest turnovers under CRS with an average efficiency score of $\bar{\theta}_{CRS} = 0.94432$ and standard deviation $\sigma_{CRS} = 0.07797$. The models also identified 11 efficient DMUs when VRS is considered, and the average efficiency score under VRS is $\bar{\theta}_{VRS} = 0.95368$, with a standard deviation of $\sigma_{VRS} = 0.07343$. The 20 stores with the lowest turnovers have an efficiency score of $\bar{\theta}_{CRS} = 0.68963$ and $\bar{\theta}_{VRS} = 0.92593$ under CRS and VRS, respectively. The standard deviations under CRS and VRS are $\sigma_{CRS} = 0.24639$ and $\sigma_{VRS} = 0.10849$, respectively. The sales patterns for replenishment products remains fairly constant throughout the year, so it is expected that the average efficiency of replenishment products is higher than for stores with products of season W16, since W16 products are older fashion items being sold out of its peak season and with a limited number of stock available.

Store ID	Turnover (in R)	θ_{CRS}	θ_{VRS}
R000338	140 629.84	1.000 00	1.000 00
R008102	136 451.95	1.000 00	1.000 00
R008227	118 331.22	1.000 00	1.000 00
R008438	107 022.95	1.000 00	1.000 00
R000211	106 593.50	1.000 00	1.000 00
R006247	101 343.87	0.986 56	0.999 20
R008727	93 836.33	0.995 97	0.997 44
R000607	87 519.99	0.931 01	0.933 82
R000551	79 416.63	1.000 00	1.000 00
R006456	78 221.99	0.829 30	0.834 97
R000210	78 048.60	0.865 03	0.891 71
R000428	76 008.10	1.000 00	1.000 00
R008063	72 269.21	0.959 60	0.966 35
R000982	71 773.26	1.000 00	1.000 00
R008767	71 225.57	0.808 81	0.809 95
R008499	70 419.44	1.000 00	1.000 00
R008430	68 334.41	0.993 64	1.000 00
R000345	67 279.02	0.800 63	0.820 56
R008272	66 735.93	0.799 37	0.819 54
R006484	66 638.86	0.916 43	1.000 00

TABLE 6.17: The efficiency scores of the top 20 turnover stores of replenishment products under CRS and VRS.

Store ID	Turnover (in R)	θ_{CRS}	θ_{VRS}
R000594	1 710.01	0.602 94	1.000 00
R000228	1 707.55	0.480 85	1.000 00
R008388	1 702.58	0.614 94	1.000 00
R000160	1 670.40	0.538 72	0.978 61
R008263	1 655.60	1.000 00	1.000 00
R006708	1 591.61	1.000 00	1.000 00
R008673	1 560.93	0.553 73	0.920 14
R006568	1 558.66	1.000 00	1.000 00
R008404	1 558.19	0.482 41	0.911 22
R000598	1 546.22	1.000 00	1.000 00
R008056	1 544.76	0.636 91	0.917 55
R008042	1 521.05	0.351 02	0.900 30
R000341	1 468.43	0.342 46	0.865 71
R000507	1 416.62	0.952 14	1.000 00
R008059	1 403.85	0.628 42	0.834 88
R006473	1 236.23	0.352 59	0.733 36
R006638	1 217.35	0.664 89	0.866 24
R006426	1 161.44	1.000 00	1.000 00
R006516	977.82	0.590 61	0.590 61
R006670	705.01	1.000 00	1.000 00

TABLE 6.18: The efficiency scores of the bottom 20 turnover stores of replenishment products under CRS and VRS.

Regional performance of stores

The average performance of stores within each region for replenishment products, as well as the number of stores that performed efficiently according to DEA under CRS and VRS respectively, is shown in Table 6.19. The efficiency scores for each store within each region is shown in Tables B.31–B.45.

Region	Number of stores	$\theta_{CRS} = 1$	$\bar{\theta}_{CRS}$	$\theta_{VRS} = 1$	$\bar{\theta}_{VRS}$
Swaziland	20	8	0.942 95	12	0.959 14
Southern Namibia	39	8	0.783 07	8	0.799 33
Northern Namibia	52	11	0.816 92	13	0.852 64
Botswana	69	6	0.879 22	7	0.881 95
Cederberg	85	7	0.736 09	16	0.773 22
Emfuleni	86	6	0.780 16	10	0.809 25
Kwena	86	9	0.842 68	9	0.844 63
Langeberg	86	5	0.615 47	9	0.659 77
North West	88	6	0.858 88	11	0.888 04
Free State	91	8	0.814 56	18	0.847 51
Lesedi	91	9	0.814 88	17	0.854 62
Gauteng	98	11	0.835 62	18	0.859 36
Limpopo	98	8	0.841 00	18	0.886 80
Thekwini	105	7	0.822 09	16	0.860 46
Tugela	107	6	0.763 35	20	0.861 95

TABLE 6.19: Average efficiency scores of stores in regions of replenishment products under CRS and VRS.

The sales patterns for replenishment products remains fairly constant throughout the year, so the inflow of products is regular and replenishment products are seldom offered at a discounted price. This explains the higher average efficiency scores when compared to the average efficiency of stores with season W16 products. However, the average efficiency scores for replenishment products is not as high as that of stores of season W17, which experiences peaks of performance in the relevant year.

6.2 DEA for subclass DMUs

DEA was run on the subclasses of the Retailer to determine how well a subclass is performing relative to other subclasses. A class of product is a type of product like jeans or tops, and subclasses are variations of that product. These subclasses come in many different styles and colours. There are 64 subclasses represented in the data, some of which span over different seasons. The number of subclasses within each product class is shown in Figure 6.1.

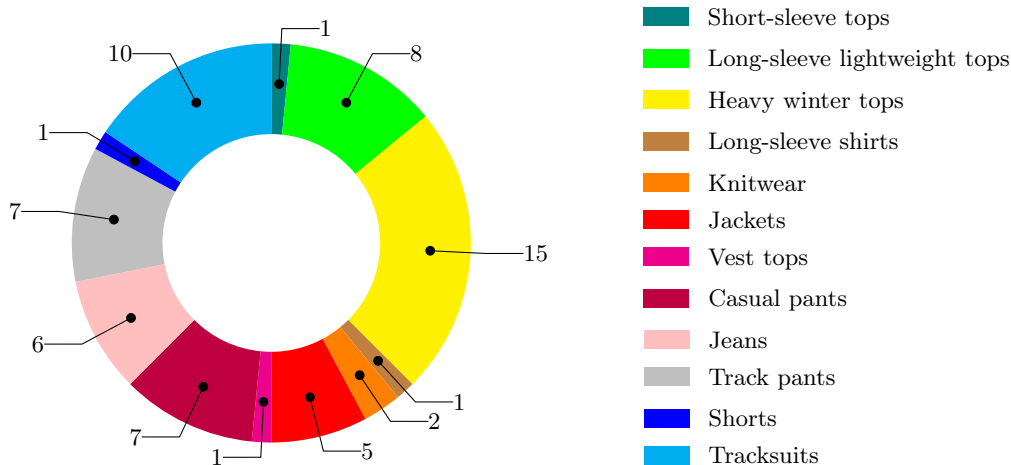


FIGURE 6.1: The number of subclasses within each product class.

6.2.1 Subclasses of season W17

The model considered 44 subclasses of products from season W17. These subclasses will be considered as DMUs for DEA, and satisfies the criteria for the number of DMUs suggested by literature in § 2.5. This allows DEA to have discriminatory power when determining the efficiency score of each of the DMUs. The efficiency scores of each of the subclasses from season W17 under CRS and VRS is given in Table 6.20.

Subclass ID	θ_{CRS}	θ_{VRS}	Subclass ID	θ_{CRS}	θ_{VRS}
W170180	0.823 05	0.833 42	W175511	0.999 59	1.000 00
W170181	0.731 07	0.989 72	W175786	0.714 06	0.876 11
W170182	0.649 80	0.920 60	W175787	0.777 69	0.850 89
W170185	0.897 62	0.979 80	W175875	0.634 98	0.938 09
W170187	0.746 69	0.898 00	W176186	0.614 47	0.980 58
W170188	0.898 57	0.952 99	W176187	1.000 00	1.000 00
W170198	0.833 07	0.985 33	W176188	0.742 46	1.000 00
W170205	1.000 00	1.000 00	W176189	0.926 59	0.969 75
W170213	1.000 00	1.000 00	W177669	1.000 00	1.000 00
W170220	1.000 00	1.000 00	W177670	0.838 27	0.886 81
W170222	1.000 00	1.000 00	W177719	1.000 00	1.000 00
W170224	0.491 33	0.651 00	W178111	0.459 38	0.468 93
W170225	0.735 66	0.885 01	W178116	0.758 07	1.000 00
W170226	0.941 93	1.000 00	W178117	0.191 09	0.191 17
W170237	0.925 24	1.000 00	W178118	0.855 84	1.000 00
W170238	0.918 61	1.000 00	W178123	0.973 37	1.000 00
W170242	1.000 00	1.000 00	W178148	1.000 00	1.000 00
W174630	0.845 92	0.862 30	W178149	0.384 99	0.421 50
W174672	0.809 36	0.926 64	W178170	0.952 39	1.000 00
W174678	0.903 18	1.000 00	W178208	1.000 00	1.000 00
W175023	0.836 77	1.000 00	W178229	0.165 74	0.169 72
W175480	0.844 46	0.924 79	W178391	0.126 37	0.129 36

TABLE 6.20: Efficiency scores of W17 subclasses under CRS and VRS.

DEA identifies 10 DMUs that performed efficiently from the calculation group of 44 DMUs when the efficiency under CRS is evaluated. The DMUs achieved an average efficiency score, which will be denoted as $\bar{\theta}_{CRS}$, of $\bar{\theta}_{CRS} = 0.79427$, with a standard deviation of $\sigma_{CRS} = 0.23220$. DEA was also run under VRS to compare the results. There are 21 DMUs identified as efficient when DEA is evaluated under VRS, and the average efficiency score, to be denoted as $\bar{\theta}_{VRS}$, is $\bar{\theta}_{VRS} = 0.87938$, with a standard deviation of $\sigma_{VRS} = 0.23358$. This is expected, since the efficient frontier takes NDRS and NIRS into account. This allows more DMUs to lie on the efficient frontier, hence more DMUs will be relatively efficient.

Some DMUs will be closer to the adjusted efficient frontier which will deem them more efficient by VRS. The standard deviation gives an indication of the spread of the efficiency scores within the calculation group. A standard deviation value closest to zero indicates that the efficiency scores of the considered DMUs are similar to that of the average efficiency score of the calculation group. This explains why the standard deviation under VRS is closer to zero than when CRS is considered. A higher average efficiency score implies a lower standard deviation.

Subclass W176186 from the calculation group of subclasses has benefited the most from DEA under VRS compared to CRS, where the efficiency score has increased by 0.366 when DEA under VRS is observed. It is important to note that this subclass still remains inefficient under both returns to scale. It is also evident that stores that are performing efficiently under CRS remained

efficient when VRS are considered. This is expected, since these DMUs are still operating at a desirable efficiency level compared to other DMUs.

There are 11 more units considered to be efficient when considering VRS rather than CRS. This implies that DMUs that are not efficient under either RTS is certainly inefficient, due to the overestimating or “conservative” nature of DEA. The efficiency scores either remained the same or increased for all DMUs when considering VRS over CRS. Again, this is expected since the efficiency frontier envelops the DMUs more closely than under CRS. The overall average of efficiency scores also improved by 0.085, but there is the possibility of improvement.

6.2.2 Subclasses of season W16

The model considered 39 subclasses of products from season W16. The efficiency scores of each of the subclasses from season W16 under CRS and VRS is given in Table 6.21.

Subclass ID	θ_{CRS}	θ_{VRS}	Subclass ID	θ_{CRS}	θ_{VRS}
W160180	1.000 00	1.000 00	W164840	1.000 00	1.000 00
W160181	0.715 29	1.000 00	W165023	0.663 78	1.000 00
W160182	0.609 08	0.993 89	W165250	1.000 00	1.000 00
W160185	0.677 87	1.000 00	W165480	0.603 19	1.000 00
W160186	0.152 99	0.237 48	W165511	0.656 60	0.899 28
W160187	0.385 53	0.682 05	W165786	0.598 65	0.889 60
W160188	0.511 85	0.912 59	W165787	0.724 52	0.820 24
W160197	0.298 33	0.755 93	W165807	0.263 66	0.410 24
W160198	0.648 56	0.988 64	W165875	1.000 00	1.000 00
W160202	1.000 00	1.000 00	W166186	0.618 29	0.872 09
W160205	1.000 00	1.000 00	W166187	0.422 26	0.538 51
W160222	0.342 19	0.698 27	W166188	0.442 71	0.585 80
W160224	0.779 40	0.779 40	W166189	0.782 20	1.000 00
W160225	0.476 08	0.541 68	W167669	0.606 64	0.766 17
W160226	0.493 21	0.918 53	W167670	0.685 19	0.918 06
W160237	0.273 17	0.835 05	W167719	0.783 51	1.000 00
W160238	1.000 00	1.000 00	W167887	1.000 00	1.000 00
W160239	0.863 08	0.902 14	W168116	0.732 04	0.922 86
W164630	1.000 00	1.000 00	W168391	0.269 31	0.269 31
W164678	0.788 07	1.000 00			

TABLE 6.21: *Efficiency scores of W16 subclasses under CRS and VRS.*

DEA identified 9 DMUs that performed efficiently from the sample of 39 DMUs for the season W16 under CRS. The average efficiency score for DMUs under CRS is $\bar{\theta}_{CRS} = 0.66326$, with a standard deviation of $\sigma_{CRS} = 0.25030$. Comparatively, DEA under VRS produced an average efficiency score of $\bar{\theta}_{VRS} = 0.84969$ with 16 DMUs performing efficiently, with a standard deviation of $\sigma_{VRS} = 0.20942$.

The efficiency score of subclass W168391 remained unchanged when considering CRS and VRS. This can happen when the ERS of an inefficient unit remains unchanged, which means that the DMUs that are directly efficient to subclass W168391 remain unchanged. The average efficiency scores are lower for season W16 subclasses compared to W17. This can be attributed to lower inputs and outputs because the products are from an older season. The discriminatory power of DEA is less because the calculation group size is smaller than the calculation group for W17, but the number of DMUs still satisfies the criteria suggested by literature in § 2.5.

6.2.3 Subclasses of replenishment products

The model considered 12 subclasses of replenishment products. The **store service level** output for all replenishment products is zero. The number of DMUs thus does not satisfy Bowlin's criteria for a large enough calculation group size as is suggested by literature in § 2.5, but satisfies all other criteria from suggested literature [74]. This is true even when considering that there is one less output as a result of **store service level** being zero. The discriminatory power in the model thus does not play as large a role since there are fewer DMUs under consideration. This means that since there are fewer DMUs that can be used for benchmarking in DEA, the scores are less accurate. The larger the size of the calculation group of DMUs considered has a direct influence on the accuracy of the efficiency scores by discriminatory power. DEA may still be run on this calculation group. The efficiency scores of each of the subclasses for replenishment products under CRS and VRS is given in Table 6.22.

Subclass ID	θ_{CRS}	θ_{VRS}
R000172	1.000 00	1.000 00
R000177	0.715 73	0.716 52
R000186	0.136 70	0.166 25
R000225	1.000 00	1.000 00
R000232	0.327 36	0.327 47
R004672	0.675 00	0.675 80
R005250	1.000 00	1.000 00
R005468	0.790 96	0.798 82
R006188	1.000 00	1.000 00
R006397	1.000 00	1.000 00
R007037	1.000 00	1.000 00
R008700	1.000 00	1.000 00

TABLE 6.22: Efficiency scores of replenishment subclasses under CRS and VRS.

DEA under CRS and VRS determined that 7 service units performed efficiently from a sample of 12. The average efficiency score for DMUs under CRS is $\bar{\theta}_{CRS} = 0.80381$ and standard deviation $\sigma_{CRS} = 0.29634$, and the efficiency score under VRS is $\bar{\theta}_{VRS} = 0.80707$ and standard deviation $\sigma_{VRS} = 0.29025$. The percentage of DMUs that are identified as efficient for DMUs is because of the number of DMUs in the sample. There is also one less output compared to the DEA models for fashion products, which will increase the discriminatory power of the DEA models. However, there are fewer DMUs considered in the models, which means the discriminatory power is less and more DMUs are identified as efficient. This is also the reason why the difference between the two scores (under CRS and VRS) is so small, with the biggest change is for subclass R000186 with an improvement of 0.0295 in the efficiency score.

6.3 Summary of results

It has been discussed earlier in this thesis that there are concerns when choosing DEA as a benchmarking tool rather than regression. However, from the statistical analysis of the results obtained in § 6.1.1, it is determined that there is merit in using DEA, for the simple fact that the influence of inputs and outputs must be considered together. While the regression models have proved that there is significance of the use of inputs and outputs, the statistics which test the accuracy of the regression models indicate that the efficiency scores cannot rely on input or output variables in isolation. It is also noted that when benchmarking against a regression line, the benchmarking is done relative to an average and not a best-practice service unit. This study

is concerned with identifying ways in which inefficient units can produce at their best efficiency, and not relative to an average performance level.

The performance of season W17 products on a store level had the better average efficiency score compared to products of season W16 and replenishment products. The freshness and the availability of these products may have an influence on the efficiency scores. These products are less likely to be sold at a discounted price and there is more of these products available than products from older seasons. Subclasses of season W17 are performing similarly to the efficiency on a store level. In both cases, there are certain DMUs that can be identified as not performing well at all, and should therefore be given specific attention when forecasting for the next season, since future products are based on the current seasons' products and performance.

The performance of season W16 products is less desirable than that of season W17 products. The average efficiency scores for W16 products, on a store-level and on a subclass-level, were less than the scores investigated for season W17 and replenishment products. The average performance of these products were expected to be better, considering that products from this season constitute 29% of the data entries and make up 39 out of the 64 subclasses of products contained in the data. The reason for the low efficiency may be because the season W16 products are less current and fashionable than that of W17 products. More than 50% of the sales of products from W16 were at promotional or discounted prices, and made less sales than replenishment and season W17 products (refer to Table 4.7).

The performance of replenishment products on a store-level is not as good as the performance of the fashion items from W17. This may be because of the life cycle of fashion items relative to the life cycle of replenishment items. Fashion products, like that of season W17, experience a peak of sales performance in the early stages of the sale season. Comparatively, the life cycle of replenishment items remains fairly constant, which may explain the average efficiency of DMUs not being very high. The average performance of subclasses, on the other hand, are much higher in average efficiency than the other subclass groups that were run. This is because there were fewer DMUs considered compared to W16 and W17 subclasses. The discriminatory power for this set of DMUs is very small, since the number of DMUs considered is less than three times the sum of the number of input and output variables, which can lead to biased inferences regarding this particular grouping.

The efficiency of the top and bottom 20 turnover stores yielded interesting results. The results showed that turnover, which is often used in ratio analyses as a performance measure, may not be looked at in isolation, which is often done in traditional performance measurement. One would expect that stores with the turnover would always be efficient and, while it may be a good indicator for performance, it was not the case for all of the calculation groups investigated in this thesis. It must be noted that stores with a range of different turnover values outperformed other DMUs in terms of efficiency. This emphasises the fact that, as was confirmed with the investigation with regression analysis, multiple inputs and outputs should all be considered when determining productivity amongst service units.

The efficiency scores of service units of the same store format and within the same regions are provided in Tables A.1–A.13 and Tables B.1–B.45, respectively. These groupings exist throughout with the assumption that DMUs within these groups are characterised similarly, and hence provides a more accurate comparison between DMUs. The goal is that these groups collaborate and learn efficient practices from similar DMUs and adopt best-practice policies for the future.

The efficiency reference set of DMUs may be applied to all groups, and under different returns to scale. The important information that is returned by the DEA models are the efficiency scores

and the weights of the ERS. The efficiency score provides information on the best-practice DMUs and which service units are performing inefficiently relative to the best-practice units. The weights of the ERS provide detail as to which DMUs the considered DMU is directly inefficient to, and what should the DMU look like, by applying the equations for projection onto the efficient frontier given in Chapter 3.

The efficiency scores under constant returns to scale is the same for both the input- and output-oriented models, since proportionality is implied. This CCR model is the most commonly used model for DEA analysis. The DEA models under VRS, however, represents a frontier that considers the different production functions of DMUs. The decision of which model to use depends on the decision maker.

CHAPTER 7

Conclusion

Contents

7.1 Thesis summary	77
7.2 Summary of findings	78
7.3 Recommendations	79
7.4 Further research	80
7.5 Achievement of objectives	80

This chapter begins with a summary of the thesis in § 7.1 and is followed by a summary of the conclusions made in this study in § 7.2. Recommendations are made based on the obtained results in § 7.3, and topics and suggestions for future research is provided in § 7.4. The thesis concludes with the objectives identified in § 1.9 which are reviewed in § 7.5.

7.1 Thesis summary

Chapter 1 contains an introduction to the retailing industry and the retail supply chain in § 1.1. The distribution network of the Retailer and the stages of planning for the Retailer is provided in § 1.2 and § 1.3. The concept of productivity as it pertains to this thesis is described in § 1.4, and benchmarking is briefly detailed in § 1.5. The basic concepts of data envelopment analysis is given in § 1.6. The problem statement to the thesis and the scope of the study is provided in § 1.7 and § 1.8, respectively. Chapter 1 then concludes with the objectives for this study in § 1.9 and the structure of the thesis in § 1.10.

Chapter 2 contains information on the relevance of efficiency analysis in modern day retailing, and how a particular approach, DEA, has been developed into multiple models and variations to determine efficiency. Industries can benefit from efficiency analysis as an additional tool to traditional performance measures to make more informed decisions. The importance of what is considered in DEA models was also discussed, as the choice of inputs and outputs, for example, must be carefully selected and must be relevant to and in line with the goals of the firm. Studies from literature that were and were not considered for this study are summarised in this chapter, as well as the reasons for choosing DEA as the appropriate tool for this study.

The mathematical formulations of the DEA models are explained in Chapter 3. The derivations of the formulae used in the model, as well as the interpretation of the resulting efficiency scores and ERS weights are shown in § 3.1. Efficiency reference sets as they relate to input and output slacks are discussed in § 3.2. DEA under constant returns to scale and variable returns to scale are both considered in this thesis, and the formulations and interpretations of each orientation of CRS and VRS are detailed in § 3.3 and § 3.4, respectively. This explanation is also detailed by means of an example to demonstrate the frontier and envelope in DEA.

Chapter 4 contains an introduction to the data, with comprehensive focus on the relevant datasets. The Retailer records data of a high standard considering the magnitude of stores and products that are received and sold each week. The nature of the data are described in § 4.1, which are the attributes of the data, the locations of stores and the dataset itself. Data validation follows in § 4.2, where the data are inspected and cleaning of any kind is recorded and accounted for. The relevant datasets for this thesis are described in detail in § 4.3. These datasets are the information that is used to determine the efficiency for the DEA models, or to calculate the inputs and outputs used in these models.

The inputs and outputs, as well as the calculation groups of DMUs for DEA are discussed in detail in Chapter 5. The calculation groups allow for the grouping of products with similar attributes, which contributes to the discriminatory power of the results. This is explained in detail in § 5.1. The decision of what components to consider in the DEA models also determines the outcome of the results. The Retailer is rich in data, so it is important to identify inputs and outputs to reflect the goals of the Retailer, and to find a result that aids the Retailer in making decisions in the future. The inputs and outputs are calculated depending on the calculation groups, and the calculation groups are used for comparability and to test efficiency scores of samples with varying sizes. A description of the outputs and the inputs considered for the DEA models is given in § 5.2 and § 5.3, respectively.

Chapter 6 contains the results from applying the inputs, outputs and calculation group criteria to the DEA models under constant and variable returns to scale. The results of DEA when considering stores as DMUs is detailed in § 6.1, and the results are calculated for different seasons, different regions and also for the different store fixture groups. All of the calculation groups considered varied in size and attributes. The results from the different groups allows for comparability of the efficiency scores. The results of DEA when subclasses are considered as decision-making units is explained in § 6.2 for all the seasons considered in this thesis. This study considered data from 1 207 different stores and the DEA scores were all obtained in SAS Enterprise Guide. The models may be adapted to include additional inputs and outputs, or to include additional DMUs.

7.2 Summary of findings

Individual efficiency scores change when tested under varying criteria, particularly when the size of the calculation group was changed. It is recommended that groups sizes are at least as large as three times the sum of the number of inputs and outputs. The larger the number of DMUs considered at once, the better the discriminatory power of the model, which leads to more accurate efficiency scores. As a result, the proportion of efficient DMUs out of the calculation group tends to be smaller when a large number of DMUs are in the calculation group compared to a smaller calculation group. A smaller sample size allows more DMUs to be identified as efficient relative to their peers. For example, the number of efficient DMUs when considering stores of season W17 under CRS is 11 out of 1 201 stores, which is 0.916% of stores that are efficient when evaluated by DEA. When only the top 20 stores with the highest turnover from that sample of 1 201 stores are considered, the number of efficient DMUs is 10 stores out of the sample of 20, which is 50% of the calculation group that is performing efficient relative to the other DMUs. This also leads to an improvement of the average efficiency scores from 0.7383 to 0.9803, which is an 0.242 increase on the average efficiency score of a DMU. It is recommended to use larger calculation groups whenever possible to avoid distorting inferences.

The decision of which returns to scale to consider is also important when using DEA as a

benchmarking model. CRS is the more discriminating of the two RTS, since the scores are more conservative. A DMU that is identified as efficient under CRS will be identified as efficient under VRS. The efficiency scores under VRS are always greater or equal to the efficiency scores under CRS when considering the same calculation group, since the concavity of the efficient frontier allows DMUs to be closer to the frontier, and therefore relatively more efficient than under CRS. This can be seen in all of the results obtained in this thesis. There are some cases where the difference between the scores is small, if not negligible. The initial result for the Retailer is whether a DMU is efficient or not by simply determining if the efficiency score is 1 or not. The second result is to analyse the ERS of each inefficient unit and employ best practices from the efficient DMUs in the ERS into the inefficient DMU.

Another important result is that financial indicators or measures in isolation does not imply efficiency. This is proved when the DEA models was run on the top and bottom stores in terms of turnover. The results show that service units with higher income may not always perform efficiently relative to other DMUs. Efficiency when determined with DEA is dependent on the mixture of the inputs and outputs, and all of these factors contribute to the performance of a DMU. It is important that emphasis is placed on the input and output mix whenever a DEA model is considered, and that the data are of a high standard.

The results from fashion and replenishment products yield different results depending on the freshness of the product. The average efficiency scores from products from season W17 and replenishment products where higher than the average efficiency of products from W16. This is due to less inputs and outputs employed into the older products of W16 as opposed to products from W17. The full price percentage for products of season W16, for instance, is greater than that of replenishment products and products from the newer season, as more products of W16 are sold at a discounted price in comparison. This is true for both cases when considering subclasses and stores as DMUs for the DEA models.

7.3 Recommendations

DEA proves itself to be a good diagnostic tool when it comes to the improvement of productivity [30]. The individual diagnosis for each service unit should be investigated, and it is recommended that attention be given to DMUs that achieve an efficiency below 1 in both constant and variable returns to scale. The two returns to scale should be considered simultaneously all the time to identify if efficiency is a result of poor performance or due to scale. Special attention should be given to DMUs with efficiency scores closer to zero than to one under both RTS.

It is also recommended that the results of all the products from multiple departments be considered at once. The baby boys' outerwear department is a very select group of products, and may not be an accurate representation of the performance of stores. Therefore, as more information is made available, a holistic benchmarking analysis on all products is recommended.

It is recommended that, although the performance is comparatively poorer than other calculation groups, the productivity of products from season W16 (or any older products) not be actively improved. The older that stock becomes, the less efficient the performance of these products are, relative to other seasons' products. There is also less stock available, so all attention to productivity improvements should be invested into newer products. The attention should be on the improvement of performance of newer products, and using this information to make planning decisions for future service units. Substantial emphasis on the performance of new seasons' products and replenishment products should be considered, since the performance of the latest styles are used for future planning seasons, and DEA will help identify the stores or subclasses

where productivity can be improved. Attention should be given to DMUs of replenishment products, because the life cycle is not as volatile as for fashion items, and diagnosis of DMUs with replenishment products allows for productivity improvements in future.

7.4 Further research

Research may be continued into efficiency for the Retailer with data from other departments. The scope is limited to the baby boys' outerwear department, which excludes a large consumer group from the experiment. Further investigation could be done by taking all products and all consumer groups into consideration. DEA can also be run on various departments and best-practices from efficient departments throughout the company can be adopted by inefficient departments.

The input/output mix will always be an important aspect when considering DEA as an efficiency measure. Further research into different inputs and outputs may be explored. Labour hours, square space of facilities and number of employees or staff employed to a specific DMU are examples of inputs that may be considered should the data become available. Potential outputs may include return on equity and investments, sales stability and stock on hand. Calculation groups also contribute to the success of DEA. Potential calculation groups can be geographical climates and store density, which refers to the proximity of stores to one another. Both of these calculation groups may indicate the type of products that are available in stores.

With the availability, it would be possible to do window analysis on the relevant data. A window analysis performs DEA for time series data to see the change in efficiency over time [24]. A DMU's performance in a specific time period is compared with its performance in other periods, as well as comparing its performance to other DMUs. It uses a moving average analogue, in which a DMU in each time period is treated as a "different" DMU. The window analysis technique is an area in literature which needs extensive contributions, but can provide insight into efficiency over time [24].

7.5 Achievement of objectives

Ten objectives were set in § 1.9 to investigate efficiency for the Retailer. Objective I (a) is achieved in Chapters 1 and 2, where the importance of efficiency is mentioned, and the relevance to benchmarking for companies in any industry plays a vital role in decision-making. Objective I (b) is addressed in Chapter 3. The formulations of a DEA model that is relevant to this thesis is described in depth in Chapter 3, including an example for illustrative purposes.

The collection of relevant data for DEA from the Retailer in Objective II (a) and II (b) is reached in Chapter 4. The data are obtained from the Retailer, and thorough validation and cleaning is completed to implement into the DEA models for sound results. The description of inputs and outputs, along with the different grouping criteria and calculation groups that are relevant to this thesis is provided in Chapter 5. This chapter provides the reasons for the choice of calculation groups which is given in § 5.1, as well as information on the calculation of the outputs and inputs given in § 5.2–5.3. Chapter 5 satisfies the specification of variables and calculation groups for DEA in Objectives III (a)–(c).

The results from the DEA are given throughout Chapter 6, which leads to the achievement of Objective IV (a). DMUs that are efficient or inefficient are portrayed in Chapter 6 as an

efficiency score of 1 or otherwise, respectively. Further results are provided in Tables A.1–A.13 and Tables B.1–B.45, which achieves Objective IV (b). Objective IV (c) is accomplished in § 6.1.1 and § 7.2, where any trends in efficiency are identified and conclusions on the DEA results are made.

List of references

- [1] ALON I, QI M and SADOWSKI RJ, 2001, *Forecasting aggregate retail sales:: a comparison of artificial neural networks and traditional methods*, Journal of Retailing and Consumer Services, **8(3)**, pp. 147–156.
- [2] ANDERSEN P and PETERSEN NC, 1993, *A procedure for ranking efficient units in data envelopment analysis*, Management science, **39(10)**, pp. 1261–1264.
- [3] AVKIRAN NK, 2001, *Investigating technical and scale efficiencies of Australian Universities through data envelopment analysis*, Socio-Economic Planning Sciences, **35(1)**, pp. 57 – 80.
- [4] BADENHORST-WEISS J, VAN BILJON E and AMBE I, 2017, *Supply chain management: A balanced approach*, 2nd Edition, Van Schaik Publishers, Pretoria.
- [5] BAI X, LV H, YIN W, DONG J and CHEN G, 2008, *Analysis on merchandise hierarchy via clustering retail records*, IEEE International Conference on Service Operations and Logistics, and Informatics, **2**, pp. 2623–2628.
- [6] BANKER RD, CHARNES A and COOPER WW, 1984, *Some models for estimating technical and scale inefficiencies in data envelopment analysis*, Management science, **30(9)**, pp. 1078–1092.
- [7] BENICIO J and DE MELLO JCS, 2015, *Productivity analysis and variable returns of scale: DEA Efficiency frontier interpretation*, Procedia Computer Science, **55**, pp. 341–349.
- [8] BERMAN B, EVANS JR and CHATTERJEE P, 1995, *Retail management: A strategic approach*, 10th Edition, Prentice Hall Englewood Cliffs (NJ).
- [9] BERRY WD, 1993, *Understanding regression assumptions*, volume 92, Sage Publications, Thousand Oaks, California (FL).
- [10] BLACKBURN V, BRENNAN S, RUGGIERO J *et al.*, 2014, *Nonparametric Estimation of Educational Production and Costs Using Data Envelopment Analysis*, volume 214, Springer Science & Business Media, New York (NY).
- [11] BLEISCHWITZ R, 2011, *Resource efficiency: Five governance challenges toward a green economy*, Journal of Industrial Ecology, **15(5)**, pp. 644–646.
- [12] BOUSSOFIANE A, DYSON RG and THANASSOULIS E, 1991, *Applied data envelopment analysis*, European Journal of Operational Research, **52(1)**, pp. 1–15.
- [13] BUSINESSDICTIONARYCOM WEBFINANCE, INC, [Online], [Cited 8 October 2018], 2018, “Inputs”, Available from <http://www.businessdictionary.com/definition/inputs.html>.
- [14] CANT C, CANT M, NEL D and STANFORD C, 2010, *Introduction to retailing*, Juta and Company Ltd, South Africa.

- [15] CHARNES A, COOPER W, WEI QL and HUANG Z, 1989, *Cone ratio data envelopment analysis and multi-objective programming*, International Journal of Systems Science, **20(7)**, pp. 1099–1118.
- [16] CHARNES A, COOPER WW, GOLANY B, SEIFORD L and STUTZ J, 1985, *Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions*, Journal of Econometrics, **30(1-2)**, pp. 91–107.
- [17] CHARNES A, COOPER WW and RHODES E, 1978, *Measuring the efficiency of decision making units*, European Journal of Operational Research, **2(6)**, pp. 429–444.
- [18] CHARNES A, COOPER WW, SEIFORD L and STUTZ J, 1982, *A multiplicative model for efficiency analysis*, Socio-Economic Planning Sciences, **16(5)**, pp. 223–224.
- [19] CHARNES A, COOPER WW, SEIFORD L and STUTZ J, 1983, *Invariant multiplicative efficiency and piecewise Cobb-Douglas envelopments*, Operations Research Letters, **2(3)**, pp. 101–103.
- [20] CHOI TM, HUI CL and YU Y, 2013, *Intelligent Fashion Forecasting Systems: Models and Applications*, Springer Science & Business Media, New York (NY).
- [21] COELLI TJ, RAO DSP, O'DONNELL CJ and BATTESE GE, 2005, *An introduction to efficiency and productivity analysis*, 2nd Edition, Springer Science & Business Media, New York (NY).
- [22] COOK WD, TONE K and ZHU J, 2014, *Data envelopment analysis: Prior to choosing a model*, Omega, **44**, pp. 1–4.
- [23] COOPER WW, SEIFORD LM and TONE K, 2006, *Introduction to data envelopment analysis and its uses: with DEA-solver software and references*, Springer Science & Business Media, New York (NY).
- [24] COOPER WW, SEIFORD LM and ZHU J, 2011, *Handbook on data envelopment analysis*, 2nd Edition, Springer Science & Business Media, New York (NY).
- [25] CORDEIRO J, SARKIS J, VAZQUEZ-BRUST D, FRATER L and DIJKSHOORN J, 2012, *An evaluation of technical efficiency and managerial correlates of solid waste management by Welsh SMEs using parametric and non-parametric techniques*, The Journal of the Operational Research Society, **63(5)**, pp. 653–664.
- [26] CUBBIN J and TZANIDAKIS G, 1998, *Regression versus data envelopment analysis for efficiency measurement: an application to the England and Wales regulated water industry*, Utilities policy, **7(2)**, pp. 75–85.
- [27] DE SOUSA MDCS and STOŠIĆ B, 2005, *Technical efficiency of the Brazilian municipalities: correcting nonparametric frontier measurements for outliers*, Journal of Productivity Analysis, **24(2)**, pp. 157–181.
- [28] DELOITTE, 2018, *Global powers of retailing 2018: Transformative change, reinvigorated commerce*, **21**, pp. 4–37.
- [29] DEMERJIAN PR, 2017, *Calculating efficiency with financial accounting data: Data envelopment analysis for accounting researchers*, SSRN Electronic Journal, University of Washington (WA).

- [30] DEVILLE A, 2009, *Branch banking network assessment using DEA: A benchmarking analysis—a note*, Management Accounting Research, **20(4)**, pp. 252 – 261.
- [31] DONNELLAN J, 2013, *Merchandise buying and management*, 4th Edition, Bloomsbury Publishing Inc., New York (NY).
- [32] DONTU N and YOO B, 1998, *Retail productivity assessment using data envelopment analysis*, Journal of Retailing, **74(1)**, pp. 89–105.
- [33] DYSON RG, ALLEN R, CAMANHO AS, PODINOVSKI VV, SARRICO CS and SHALE EA, 2001, *Pitfalls and protocols in DEA*, European Journal of operational research, **132(2)**, pp. 245–259.
- [34] EMROUZNEJAD A, 2018, *Solutions to the DEA Model*, [Online], [Cited 8 August 2018], Available from <http://deazone.com/en/resources/tutorial/solutions-to-the-dea-model>.
- [35] FANG C, GUAN X, LU S, ZHOU M and DENG Y, 2013, *Input–output efficiency of urban agglomerations in China: An application of data envelopment analysis (DEA)*, Urban Studies, **50(13)**, pp. 2766–2790.
- [36] FÄRE R and LOVELL CK, 1978, *Measuring the technical efficiency of production*, Journal of Economic Theory, **19(1)**, pp. 150–162.
- [37] FARRELL MJ, 1957, *The measurement of productive efficiency*, Journal of the Royal Statistical Society. Series A (General), **120(3)**, pp. 253–290.
- [38] GANDHI A and SHANKAR R, 2014, *Efficiency measurement of Indian retailers using data envelopment analysis*, International Journal of Retail & Distribution Management, **42(6)**, pp. 500–520.
- [39] GANDHI A and SHANKAR R, 2016, *Strategic resource management model and data envelopment analysis for benchmarking of indian retailers*, Benchmarking: An International Journal, **23(2)**, pp. 286–312.
- [40] GIUFFRIDA A and GRAVELLE H, 2001, *Measuring performance in primary care: econometric analysis and DEA*, Applied Economics, **33(2)**, pp. 163–175.
- [41] GOLANY B, 1988, *An interactive MOLP procedure for the extension of DEA to effectiveness analysis*, The Journal of the Operational Research Society, **39(8)**, pp. 725–734.
- [42] GOLANY B and ROLL Y, 1989, *An application procedure for DEA*, Omega, **17(3)**, pp. 237–250.
- [43] GRINYER M and GOLDSMITH H, 1995, *Benchmarking for competitive advantage*, BBC for Business Publications, London.
- [44] HART C and RAFIQ M, 2006, *The dimensions of assortment: A proposed hierarchy of assortment decision making*, Int. Rev. of Retail, Distribution and Consumer Research, **16(3)**, pp. 333–351.
- [45] HARTLEY RF, 1984, *Retailing: challenge and opportunity*, 1st Edition, Houghton Mifflin School, Boston (MA).
- [46] HJALMARSSON L, KUMBHAKAR SC and HESHMATI A, 1996, *DEA, DFA and SFA: a comparison*, Journal of Productivity Analysis, **7(2-3)**, pp. 303–327.

- [47] HOCHMAN HM and RODGERS JD, 1969, *Pareto optimal redistribution*, The American Economic Review, pp. 542–557.
- [48] HOMBURG C, 2001, *Using data envelopment analysis to benchmark activities*, International Journal of Production Economics, **73(1)**, pp. 51–58.
- [49] HUGO W, BADENHORST-WEISS J and VAN BILJON E, 2010, *Supply chain management: Logistics in perspective*, Van Schaik Publishers, Pretoria.
- [50] ISMAIL M, 2004, *Dea analysis of bank performance in malaysia*, Proceedings of DEA2004, Birmingham, p. 15.
- [51] KONTODIMOPOULOS N and NIAKAS D, 2005, *Efficiency measurement of hemodialysis units in Greece with data envelopment analysis*, Health Policy, **71(2)**, pp. 195–204.
- [52] LANCHEROS J and NIETO C, 2015, *An application of the DEA optimization methodology to make more effective and efficient collection calls*, (Unpublished), SAS Institute Inc., Cary (NC).
- [53] LESEURE M, 2010, *Key concepts in Operations Management*, 1st Edition, Sage Publications Inc., London.
- [54] LEVY M, WEITZ BA and GREWAL D, 2012, *Retailing management*, volume 6, 8th Edition, McGraw-Hill/Irwin New York (NY).
- [55] LEWISON DM, 1991, *Retailing*, 5th Edition, Macmillan International Higher Education, London.
- [56] LUMMUS RR and VOKURKA RJ, 1999, *Defining supply chain management: a historical perspective and practical guidelines*, Industrial Management & Data Systems, **99(1)**, pp. 11–17.
- [57] MCCARTHY TM, DAVIS DF, GOLICIC SL and MENTZER JT, 2006, *The evolution of sales forecasting management: A 20-year longitudinal study of forecasting practices*, Journal of Forecasting, **25(5)**, pp. 303–324.
- [58] MIRDEHGHAN S, VAKILI J and NAZAARI M, 2015, *Relations among technical, cost and revenue efficiencies in data envelopment analysis*, International Journal of Applied Mathematics, **45(4)**, pp. 2–9.
- [59] MORITA H and AVKIRAN NK, 2009, *Selecting inputs and outputs in data envelopment analysis by designing statistical experiments*, Journal of the Operations Research society of Japan, **52(2)**, pp. 163–173.
- [60] MOSTAFA MM, 2009, *Benchmarking the US specialty retailers and food consumer stores using data envelopment analysis*, International Journal of Retail & Distribution Management, **37(8)**, pp. 661–679.
- [61] NATARAJA NR and JOHNSON AL, 2011, *Guidelines for using variable selection techniques in data envelopment analysis*, European Journal of Operational Research, **215(3)**, pp. 662–669.
- [62] NGOIE KJ and KOCH SK, 2005, *DEA applied to a Gauteng sample of South African public hospitals*, (Unpublished), Department of Economics, University of Pretoria Working Paper No28, Pretoria.

- [63] OREGON BUSINESS PLAN, 2018, *Industry Clusters FAQ*, [Online], [Cited 26 November 2018], Available from <https://oregonbusinessplan.org/about-the-plan/industry-clusters/industry-clusters-faq/>.
- [64] OSBORNE J and WATERS E, 2002, *Four assumptions of multiple regression that researchers should always test*, Practical Assessment, Research & Evaluation, **8(2)**, pp. 1–5.
- [65] PANZAR JC and WILLIG RD, 1977, *Economies of scale in multi-output production*, The Quarterly Journal of Economics, **91(3)**, pp. 481–493.
- [66] PARADI JC, VELA SA and ZHU H, 2010, *Adjusting for cultural differences, a new DEA model applied to a merged bank*, Journal of Productivity Analysis, **33(2)**, pp. 109–123.
- [67] PIENAAR WJ and VOGT JJ, 2014, *Business logistics management: a supply chain perspective*, 5th Edition, Oxford University Press Southern Africa, Stellenbosch.
- [68] PORTER ME, 1998, *Clusters and the new economics of competition*, volume 76, Harvard Business Review, Boston (MA).
- [69] PROJECT MANAGEMENT INSTITUTE, 2015, *A Guide to the Project Management Body of Knowledge: PMBOK Guide*, volume 3, PMI Publishing Division, Pennsylvania (PA).
- [70] RAA T, 2009, *The economics of benchmarking: Measuring performance for competitive advantage*, Palgrave Macmillan, New York (NY).
- [71] RAJARAM K, 2008, *Merchandise planning models for fashion retailing*, pp. 105–145 in *Supply Chain Analysis*, pp. 105–145. Springer Science & Business Media, New York (NY).
- [72] ROGERS D, 1992, *A review of sales forecasting models most commonly applied in retail site evaluation*, International Journal of Retail & Distribution Management, **20(4)**, pp. 1–4.
- [73] RUGGIERO J, 2010, *Frontiers in major league baseball: nonparametric analysis of performance using data envelopment analysis*, volume 1, Springer Science & Business Media, New York (NY).
- [74] SARKIS J, 2007, *Preparing your data for DEA*, pp. 305–320 in *Modeling data irregularities and structural complexities in data envelopment analysis*, pp. 305–320. Springer Science & Business Media, New York (NY).
- [75] SARKIS J and TALLURI S, 2004, *Performance based clustering for benchmarking of US airports*, Transportation Research Part A: Policy and Practice, **38(5)**, pp. 329–346.
- [76] SAS INSTITUTE, 2018, *Analytics, Business Intelligence and Data Management*, [Online], [Cited 23 May 2018], Available from https://www.sas.com/en_us/.
- [77] SAS INSTITUTE, 2017, *SAS/OR® 12.1 User's guide: Mathematical programming examples*, [Online], [Cited 5 June 2017], Available from http://support.sas.com/documentation/cdl/en/ormpex/65555/HTML/default/viewer.htm#ormpex_ex22_toc.htm.
- [78] SEIFORD LM, 1990, *Models, extensions, and applications of data envelopment analysis: A selected reference set*, Computers, Environment and Urban Systems, **14(2)**, pp. 171–175.
- [79] SHABANPOUR H, YOUSEFI S and SAEN RF, 2017, *Future planning for benchmarking and ranking sustainable suppliers using goal programming and robust double frontiers DEA*, Transportation Research Part D: Transport and Environment, **50**, pp. 129–143.

- [80] SHARMA V and CHOUDHARY H, 2010, *Measuring operational efficiency of retail stores in Chandigarh Tri-city using DEA*, Journal of Services Research, **10(2)**, pp. 99–101.
- [81] SHERMAN HD and GOLD F, 1985, *Bank branch operating efficiency: Evaluation with data envelopment analysis*, Journal of banking & finance, **9(2)**, pp. 297–315.
- [82] SHERMAN HD and ZHU J, 2006, *Service productivity management: Improving service performance using data envelopment analysis (DEA)*, Springer Science & Business Media, New York (NY).
- [83] STEWART TJ, 2010, *Goal directed benchmarking for organizational efficiency*, Omega, **38(6)**, pp. 534–539.
- [84] TANG CS, TEO CP and WEI KK, 2007, *Supply chain analysis: a handbook on the interaction of information, system and optimization*, volume 119, Springer Science & Business Media, New York (NY).
- [85] TAYLOR R, 1990, *Interpretation of the correlation coefficient: a basic review*, Journal of Diagnostic Medical Sonography, **6(1)**, pp. 35–39.
- [86] THANASSOULIS E, DYSON RG and FOSTER MJ, 1987, *Relative efficiency assessments using data envelopment analysis: An application to data on rates departments*, Journal of the Operational Research Society, **38(5)**, pp. 397–411.
- [87] THOMAS RR, BARR RS, CRON WL and SLOCUM JW, 1998, *A process for evaluating retail store efficiency: a restricted DEA approach*, International Journal of Research in Marketing, **15(5)**, pp. 487 – 503.
- [88] TOMKINS C and GREEN R, 1988, *An experiment in the use of data envelopment analysis for evaluating the efficiency of uk university departments of accounting*, Financial Accountability & Management, **4(2)**, pp. 147–164.
- [89] TONE K, 2001, *A slacks-based measure of efficiency in data envelopment analysis*, European Journal of Operational Research, **130(3)**, pp. 498 – 509.
- [90] VASSILOGLOU M and GIOKAS D, 1990, *A study of the relative efficiency of bank branches: an application of data envelopment analysis*, Journal of the Operational Research Society, **41(7)**, pp. 591–597.
- [91] WACKERLY D, MENDENHALL W and SCHEAFFER RL, 2014, *Mathematical statistics with applications*, 7th Edition, Brooks/Cole, Cengage Learning, Belmont (CA).
- [92] WIID J and DIGGINES CN, 2012, *Fundamentals of merchandising*, Juta and Company Ltd, Cape Town.
- [93] WILLIAMS MN, GRAJALES CAG and KURKIEWICZ D, 2013, *Assumptions of multiple regression: correcting two misconceptions*, Practical Assessment, Research & Evaluation, **18(11)**, pp. 1–14.
- [94] WINSTON WL, 2004, *Operations research: applications and algorithms*, 4th Edition, Brooks/Cole, Cengage Learning, Boston (MA).
- [95] YANG JB, WONG BY, XU DL and STEWART TJ, 2009, *Integrating DEA-oriented performance assessment and target setting using interactive MOLP methods*, European Journal of Operational Research, **195(1)**, pp. 205 – 222.

-
- [96] ZHU J, 2014, *Quantitative models for performance evaluation and benchmarking: data envelopment analysis with spreadsheets*, volume 213, Springer Science & Business Media, New York (NY).

APPENDIX A

Efficiency scores per store format

Appendix A is a compilation of the efficiency scores of stores per store format. The scores may differ when the calculation group considers all stores as decision-making units. For comparability, the calculation groups are divided into store formats for all products over all seasons for which data were available. The unique store ID number is provided, along with the efficiency score under constant and variable returns to scale, respectively.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
B0113	0.964306215	0.967816587	B6465	0.923221523	0.937160392
B0150	0.814402104	1.000000000	B6476	0.972637218	0.99392941
B0152	0.844350399	0.992019938	B6480	0.969362106	0.984752797
B0167	1.000000000	1.000000000	B6488	0.972222437	0.982879171
B0178	0.948267699	0.949728232	B6490	0.927829796	0.975555374
B0182	0.844476641	0.900034454	B6538	0.924149983	0.92763875
B0288	0.85016006	0.965738464	B6568	0.813986782	0.83867983
B0296	0.868413301	0.932265964	B6705	0.880034121	1.000000000
B0316	0.870490008	0.936839924	B6708	0.812302825	0.880639967
B0411	0.838322634	0.896669326	B6710	0.698332581	0.742007469
B0421	0.735431606	0.986464742	B8102	1.000000000	1.000000000
B0499	0.971177014	0.98173978	B8204	0.984477789	1.000000000
B0535	0.720476725	0.865930912	B8215	0.997113854	1.000000000
B0549	1.000000000	1.000000000	B8238	1.000000000	1.000000000
B0556	1.000000000	1.000000000	B8281	0.981971332	0.988000676
B0570	0.934501238	1.000000000	B8289	0.963043821	0.963358203
B0598	1.000000000	1.000000000	B8295	0.955725054	0.957853563
B0648	1.000000000	1.000000000	B8298	0.880045195	0.885188217
B0693	0.805370737	0.818469908	B8301	0.857493144	0.898819321
B0735	0.996640922	1.000000000	B8325	0.967672125	0.977749753
B0787	1.000000000	1.000000000	B8365	0.858639941	0.917598424
B0918	0.954660973	1.000000000	B8425	0.910543312	0.952610662
B4119	0.984098772	1.000000000	B8426	0.984431703	0.984729219
B6115	0.91777539	0.922824082	B8510	1.000000000	1.000000000
B6127	0.756648414	0.854013632	B8511	0.920374279	0.925254844
B6134	0.762109102	0.913828891	B8652	1.000000000	1.000000000
B6157	0.98776615	0.988763939	B8673	1.000000000	1.000000000
B6235	0.969177649	0.981473772	B8695	0.920354031	0.925039419
B6284	0.896113811	0.953136391	B8724	0.917478406	0.963808266
B6390	0.985002233	0.985046764	B8736	0.817926117	0.912467409
B6433	0.897238928	0.97296815	B8751	1.000000000	1.000000000

TABLE A.1: Efficiency scores of stores of store format “B” under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
C0111	0.93487672	0.962365299	C0630	0.745756271	0.816001273
C0131	0.798968859	0.905577093	C0732	0.760301546	0.896328105
C0154	0.915289957	1.000000000	C0733	0.976867119	0.999984357
C0180	0.816127937	0.865302939	C0905	0.979685994	1.000000000
C0199	0.98728516	1.000000000	C0911	0.864683322	1.000000000
C0222	0.880058747	0.904474815	C0922	1.000000000	1.000000000
C0228	0.816994763	0.84645548	C0955	1.000000000	1.000000000
C0234	0.893487805	0.996984543	C0970	0.928179979	0.975049642
C0236	0.868165387	0.969465879	C0978	0.894654468	0.905467828
C0259	0.727441719	0.932147302	C4170	1.000000000	1.000000000
C0268	0.740369146	0.818077605	C4203	0.82572525	0.847289166
C0290	0.911997989	1.000000000	C4258	0.745727454	0.901163031
C0312	0.845431662	0.901737827	C4271	1.000000000	1.000000000
C0313	0.73731178	0.840810184	C4512	0.842741849	0.877851638
C0325	0.853604332	0.984257626	C6104	1.000000000	1.000000000
C0328	0.936211376	0.949566317	C6116	0.793956072	0.814960379
C0335	0.859610486	0.891784269	C6119	0.91469807	0.944660845
C0356	0.877901645	1.000000000	C6122	0.890765706	0.892621772
C0364	1.000000000	1.000000000	C6130	0.899333621	0.968436658
C0367	0.995700583	1.000000000	C8011	0.931976013	0.984679969
C0374	0.769348413	0.809811492	C8042	0.510868533	0.611163573
C0395	0.831571966	0.881621335	C8059	1.000000000	1.000000000
C0402	1.000000000	1.000000000	C8084	0.880790001	0.882622265
C0405	0.918948984	0.982948537	C8089	0.967913722	1.000000000
C0409	0.790703308	0.829997886	C8224	0.822680795	0.823817678
C0417	1.000000000	1.000000000	C8263	1.000000000	1.000000000
C0424	0.863136768	0.875132679	C8284	0.83937052	0.854320163
C0425	1.000000000	1.000000000	C8288	1.000000000	1.000000000
C0444	0.724032095	0.833960958	C8293	0.872885599	0.908952337
C0460	0.962505588	1.000000000	C8333	0.822090959	0.83903542
C0469	0.682352864	0.771987275	C8338	0.83733467	0.914172887
C0474	0.861802365	0.891638242	C8348	0.869108058	0.869219555
C0477	0.848500341	0.854679989	C8399	0.90534844	0.939452555
C0482	0.923378411	0.950709089	C8404	0.799179668	0.875028315
C0492	0.881313995	0.959906119	C8534	0.853515653	0.890354435
C0495	0.818876416	0.874997121	C8597	0.915752344	0.986075829
C0509	0.776741686	0.817606037	C8678	0.777179481	0.887219999
C0520	0.934364914	0.941562622	C8717	0.921978462	0.929280255
C0525	0.827039924	0.918700501	C8739	0.783507098	0.852155111
C0544	1.000000000	1.000000000	C8758	0.873170613	0.942784081
C0594	0.805043558	0.901961497	C8775	0.793894887	0.803504793
C0599	0.934571871	0.945427306	C8790	0.837255751	0.900094177
C0619	0.954657835	0.963724779	C8791	0.817715152	0.879916432
C0628	0.865195967	0.96188269			

TABLE A.2: Efficiency scores of stores of store format “C” under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
D0105	0.972553387	0.987037282	D6164	0.922298007	0.953109647
D0112	1.000000000	1.000000000	D6165	0.929715267	0.939206602
D0134	0.889681415	0.97918619	D6181	0.881481132	0.88169764
D0173	0.859591253	0.961799392	D6182	0.897075247	0.961958929
D0201	0.83360226	0.893203163	D6189	0.979675451	1.000000000
D0202	0.824092225	0.920494178	D6192	1.000000000	1.000000000
D0210	0.944733901	1.000000000	D6203	0.81184237	0.914518525
D0219	0.958187664	0.965721691	D6206	0.985573519	1.000000000
D0229	1.000000000	1.000000000	D6209	1.000000000	1.000000000
D0231	0.869478865	0.961430116	D6224	0.943026438	0.947220407
D0251	1.000000000	1.000000000	D6241	0.918562489	0.941631025
D0310	0.851370879	0.905176972	D6247	1.000000000	1.000000000
D0345	0.956537533	0.963279363	D6277	1.000000000	1.000000000
D0354	0.922587546	0.923119427	D6281	0.982415462	0.984499403
D0398	0.88653296	0.90033696	D6333	0.951071238	0.984697866
D0400	1.000000000	1.000000000	D6340	0.971636916	1.000000000
D0401	0.946958816	0.978558406	D6356	0.966110134	0.993506801
D0419	0.995892401	0.996902294	D6358	0.996380786	1.000000000
D0436	0.940984107	0.946567127	D6383	0.863233235	0.916708501
D0448	1.000000000	1.000000000	D6402	0.899820548	0.952703706
D0458	0.885665881	0.88706403	D6415	0.950839372	0.962009817
D0484	1.000000000	1.000000000	D6459	0.923989929	0.93693261
D0542	1.000000000	1.000000000	D6466	0.869240374	0.905247513
D0543	0.875643037	0.886679447	D6485	1.000000000	1.000000000
D0546	0.818978852	0.889276196	D6602	0.852599978	0.917934195
D0548	0.919258048	1.000000000	D8079	0.879316854	0.923646833
D0607	0.998530577	1.000000000	D8111	0.961026477	0.979495857
D0618	0.907621491	0.913485453	D8114	0.868618394	0.924543552
D0639	0.982851174	0.996793485	D8319	0.738749179	0.798776607
D0718	0.94966223	0.972299739	D8376	1.000000000	1.000000000
D0730	0.922595208	0.932904011	D8387	0.958187124	0.986181887
D0737	0.931298685	0.943280818	D8408	1.000000000	1.000000000
D0774	0.983606142	0.98477247	D8430	1.000000000	1.000000000
D0913	0.929233633	0.93777929	D8460	0.871043864	0.909506085
D0919	0.931716896	0.997075116	D8465	0.863066059	0.937126416
D0965	1.000000000	1.000000000	D8527	0.931101525	0.935108536
D0976	0.896975037	0.923245459	D8559	0.970119287	0.970181215
D4132	0.870056481	0.90948238	D8660	1.000000000	1.000000000
D4231	0.968165923	0.969058287	D8667	0.891926768	0.936365023
D6135	0.988586689	0.989551043	D8674	0.963361177	1.000000000
D6136	1.000000000	1.000000000	D8687	0.876080095	0.931536564
D6137	0.990431739	1.000000000	D8712	0.968764764	0.990362887
D6138	0.975958955	1.000000000	D8727	1.000000000	1.000000000
D6145	0.991829332	1.000000000	D8743	0.943989919	0.978413
D6146	0.93698696	0.989936958	D8767	0.959791704	0.990274266
D6159	0.832111399	0.905949832	D8768	0.939587449	0.978807246
D6161	0.970890089	0.990403797	D8781	1.000000000	1.000000000

TABLE A.3: Efficiency scores of stores of store format “D” under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
E0144	0.988527106	1.000000000	E6543	0.874151903	0.919454103
E0217	0.90733428	0.951431373	E6545	1.000000000	1.000000000
E0274	0.885453228	0.926347416	E6549	0.881450913	0.881910563
E0303	0.951151059	0.956854013	E6550	0.938687887	0.961657686
E0323	0.983191193	0.997176267	E6557	0.890697555	0.922462373
E0339	1.000000000	1.000000000	E6558	0.951635427	1.000000000
E0357	0.952263251	0.998317541	E6559	0.839553205	0.933321749
E0359	0.836303603	0.918142149	E6561	0.954584623	0.999310818
E0386	0.974451357	1.000000000	E6562	1.000000000	1.000000000
E0408	0.856858053	0.936218268	E6563	0.951923751	0.975691969
E0418	0.881990824	0.922911572	E6565	0.882998882	0.932278002
E0457	0.870741396	0.886527598	E6569	0.879120361	0.912622542
E0467	0.840666237	0.922938365	E6576	0.836098946	0.8998467
E0473	0.840518917	0.875807128	E6578	0.906759332	0.928048467
E0494	1.000000000	1.000000000	E6579	0.927107797	0.963545575
E0505	0.997112586	1.000000000	E6580	0.962291703	0.980071839
E0610	0.977962537	1.000000000	E6581	0.913035613	0.955547031
E0614	0.970929724	0.981168765	E6582	1.000000000	1.000000000
E0640	0.858072144	0.887177576	E6585	0.883976145	0.896282495
E0736	0.934399276	0.959180984	E6586	0.903834894	0.938398362
E0792	0.943586922	0.952004034	E6591	0.939179638	0.940764313
E0907	0.921245299	0.928745112	E6593	0.924690129	0.936255538
E0924	0.903478918	0.913347533	E6594	0.982171114	1.000000000
E0936	0.935483972	0.950178574	E6596	0.911316869	0.925539225
E0982	1.000000000	1.000000000	E6598	1.000000000	1.000000000
E6102	0.849361897	0.909145188	E6599	0.938748293	0.95010972
E6117	1.000000000	1.000000000	E6605	0.982017421	0.99020416
E6124	0.889984231	0.890554169	E6606	0.890226163	0.955153135
E6201	0.952349176	0.963831141	E6609	1.000000000	1.000000000
E6286	0.964397261	0.966221653	E6613	0.904896163	0.905865644
E6324	0.917216074	0.929199247	E6614	0.91287284	0.931935791
E6378	1.000000000	1.000000000	E6615	0.924093236	1.000000000
E6438	0.832886944	0.854276858	E6616	0.881603474	0.913600693
E6487	1.000000000	1.000000000	E6619	0.90252663	0.932504784
E6494	0.951788773	0.971725339	E6623	0.873670873	0.896436348
E6502	0.854622774	0.854767034	E6624	0.932691186	0.937384515
E6505	0.818109169	0.919666614	E6626	0.906613423	1.000000000
E6506	0.890717418	0.906850955	E6629	1.000000000	1.000000000
E6507	0.951268613	0.989574735	E6631	0.909048135	0.930869409
E6510	0.929176534	0.942468694	E6632	0.880813054	0.881628788
E6511	0.946793587	0.948423056	E6637	0.953375463	1.000000000
E6512	1.000000000	1.000000000	E6649	0.947576365	1.000000000
E6513	1.000000000	1.000000000	E6651	0.96790189	0.972328881
E6515	1.000000000	1.000000000	E6657	0.953615823	0.959608647
E6518	0.979859121	0.981046137	E6658	0.935281168	0.948464815
E6520	1.000000000	1.000000000	E6663	1.000000000	1.000000000
E6522	0.917371086	0.951010626	E6666	0.921546296	0.964530116
E6526	0.9326004	0.999751717	E6668	0.997258891	1.000000000
E6527	0.86690392	0.891708448	E6670	1.000000000	1.000000000
E6529	0.947154414	0.948880692	E6674	1.000000000	1.000000000
E6531	0.825803329	0.896877887	E6677	0.893040603	0.908019686
E6533	0.885228582	0.936767099	E6678	0.738075263	0.864402146
E6534	0.978877177	0.979335066	E6681	0.993143519	1.000000000
E6537	1.000000000	1.000000000	E6691	0.887126478	0.887126478
E6542	0.978781617	1.000000000	E6696	1.000000000	1.000000000

TABLE A.4: Efficiency scores of stores of store format “E” under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}
E6701	0.701362231	0.954278427
E6702	0.903391568	1.000000000
E6709	0.603290437	0.614837633
E8023	0.823623285	0.869215796
E8098	0.937353567	0.992591402
E8261	0.83624594	0.857052109
E8272	0.941719244	0.994374183
E8356	0.968078468	1.000000000
E8370	0.92842998	1.000000000
E8384	0.923048952	0.925753324
E8420	0.908354761	0.927575227

Store ID	θ_{CRS}	θ_{VRS}
E8431	0.975900784	0.976645907
E8493	0.981526574	1.000000000
E8521	0.846856881	0.847783771
E8531	0.945810986	0.94749196
E8603	0.846443714	0.897556746
E8646	0.851025858	0.88361672
E8681	0.954384775	0.978558224
E8705	0.942674185	0.953849249
E8713	0.903225736	0.930698016
E8753	0.985030223	0.997692045
E8769	0.991380012	1.000000000

TABLE A.5: Efficiency scores of stores of store format “E” under CRS and VRS (continued).

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
F0102	0.844347084	0.91408484	F0414	0.769401897	0.820227574
F0104	0.927693192	0.9802757	F0422	0.802922551	0.814358397
F0108	0.928776842	0.969377285	F0430	0.700587756	0.82694945
F0115	0.896472634	0.919547786	F0435	0.829005532	0.830792671
F0124	0.894979687	0.895985723	F0437	0.852107853	0.852959557
F0126	0.946334634	0.985643097	F0443	0.956646364	0.958654793
F0132	0.853460161	0.89646906	F0447	0.945618064	0.945637504
F0137	0.813799595	0.926978844	F0452	0.979658266	0.985699684
F0142	0.715290368	0.78124636	F0464	1.000000000	1.000000000
F0148	0.874441758	0.972046235	F0476	0.841086005	0.884963582
F0149	0.954384893	0.959660611	F0478	0.770268142	0.896587415
F0151	0.867053326	1.000000000	F0487	0.794118618	0.898861295
F0156	0.877033486	0.911357429	F0490	0.913690901	0.916556515
F0160	0.700289325	0.711866506	F0498	0.962840867	0.974611102
F0174	0.88361959	0.929158608	F0503	0.999877243	1.000000000
F0176	1.000000000	1.000000000	F0507	1.000000000	1.000000000
F0183	0.820220333	0.952354392	F0513	0.823183682	0.89859801
F0189	0.881567877	0.912200422	F0532	0.840824907	0.894645516
F0191	0.893175522	0.920197826	F0533	0.929554687	0.931806277
F0195	0.819445706	0.932720436	F0541	1.000000000	1.000000000
F0215	0.859426715	0.88040366	F0552	0.8989197	0.915009704
F0225	0.985595247	0.995912274	F0561	0.928592162	0.952560688
F0226	0.766125916	0.93562625	F0582	0.894591263	0.899479275
F0233	0.982265326	0.999996146	F0583	0.838051903	0.872959438
F0237	0.981220991	0.983393987	F0591	0.892947689	0.906759814
F0241	0.92501884	0.978943174	F0601	0.934821888	0.993408469
F0247	1.000000000	1.000000000	F0604	0.871274519	0.871701555
F0248	1.000000000	1.000000000	F0605	0.883968883	0.913439256
F0249	0.943666033	0.958221457	F0620	0.925324336	0.947076295
F0253	0.849809098	0.95050127	F0634	1.000000000	1.000000000
F0254	0.977541343	0.985994176	F0642	0.811646944	0.819447899
F0267	0.864119874	0.967159416	F0651	0.915005234	0.955661281
F0276	0.872519024	0.922340046	F0681	0.935029876	0.93604013
F0280	0.859803773	0.947849697	F0689	0.98823044	1.000000000
F0282	0.936625402	0.936850499	F0690	0.873895668	0.931644259
F0283	0.957341162	0.991486951	F0695	0.933130089	1.000000000
F0285	0.737871647	0.870655281	F0696	0.786145642	0.865841136
F0321	0.955968125	0.972235233	F0710	0.737593179	0.756119643
F0322	0.883432332	0.942040168	F0723	0.859312905	0.91781063
F0333	0.930507435	0.955973913	F0727	0.844819591	0.953645137
F0341	0.622613912	0.737985702	F0734	0.969157436	0.984518703
F0344	0.776012297	0.864971563	F0746	0.873395948	0.874841581
F0347	0.787022915	0.817933372	F0778	0.947599601	0.952600775
F0349	1.000000000	1.000000000	F0789	0.810739088	0.915237069
F0350	0.923622261	0.950471731	F0909	0.839962976	0.877693864
F0352	0.804364382	0.898494646	F0914	0.935319933	0.966202762
F0353	1.000000000	1.000000000	F0931	0.993779549	1.000000000
F0366	0.778499929	0.915013882	F0943	0.986195149	1.000000000
F0371	0.888645942	0.930541227	F0953	0.907172926	0.91119905
F0381	0.843403013	0.863538767	F0987	0.946685077	0.966460955
F0387	0.881338505	0.919652771	F0993	0.924172336	0.943407924
F0389	0.828715851	0.847516417	F4168	0.880651773	0.882117215
F0390	1.000000000	1.000000000	F4208	1.000000000	1.000000000
F0391	0.917503241	0.950478009	F4259	0.995200005	1.000000000
F0406	0.886953456	0.941438812	F4509	0.827110113	0.828066227

TABLE A.6: Efficiency scores of stores of store format “F” under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
F6279	0.805502508	0.961529149	F8382	0.869969913	0.880778367
F6638	1.000000000	1.000000000	F8385	0.736927573	0.862198637
F8006	0.918364468	0.958901895	F8388	0.87455046	0.929132557
F8008	0.819066209	0.846771563	F8391	1.000000000	1.000000000
F8012	0.819358267	0.963849	F8398	0.938651742	0.969364026
F8018	0.834372367	0.916750215	F8402	0.828679197	0.831510509
F8024	0.943162692	0.954310915	F8427	0.847414998	0.864184614
F8035	0.954960004	0.958551913	F8434	1.000000000	1.000000000
F8045	0.846110083	0.941154806	F8462	0.939862219	0.962569087
F8046	0.88647158	0.908592977	F8476	0.992489797	1.000000000
F8056	0.907760102	1.000000000	F8486	0.897057224	0.961242098
F8065	0.920379139	0.942291841	F8488	0.824206673	0.889290271
F8067	0.883355923	0.92724316	F8489	0.868789425	0.916262892
F8073	0.865248618	0.905507479	F8495	0.879843052	0.98314805
F8082	0.879355565	0.910089752	F8497	1.000000000	1.000000000
F8092	0.705549623	0.783127073	F8509	0.769672287	0.809937955
F8096	0.917636738	0.919520569	F8517	0.902666313	0.960508903
F8099	0.950228988	0.983537895	F8520	1.000000000	1.000000000
F8113	0.890167086	0.900041013	F8529	0.971969315	1.000000000
F8128	0.825773115	0.842527739	F8536	0.909887323	0.925376179
F8139	0.877742663	0.886009223	F8537	0.946124013	0.949670687
F8202	0.783054419	0.852839508	F8546	0.967290565	1.000000000
F8212	0.814225385	0.81681367	F8553	0.799127031	0.869992052
F8220	0.814605529	0.856472999	F8558	0.890514081	0.890936531
F8223	0.813651576	0.87878324	F8591	1.000000000	1.000000000
F8225	0.93683784	0.982044948	F8600	1.000000000	1.000000000
F8226	0.974461531	0.974893471	F8649	0.895871292	0.915940573
F8239	0.854550317	0.915086493	F8665	0.825593635	0.844823456
F8247	0.855540164	0.940179293	F8670	0.957138318	0.98451658
F8251	0.818113614	0.847641621	F8672	0.696359756	0.82919479
F8268	0.662048517	0.763716466	F8677	0.883326518	0.956757863
F8273	0.818194312	0.862268175	F8696	0.904482989	0.97930728
F8280	0.927779742	0.951378621	F8726	0.956200812	1.000000000
F8287	0.826866224	0.848939575	F8728	0.890315029	0.960732033
F8291	0.755588577	0.859366423	F8731	0.803890914	0.80664759
F8292	0.881012028	0.883193629	F8732	0.882762403	0.944135998
F8303	0.896018607	0.897090087	F8735	0.962917615	0.99180983
F8310	0.880078291	0.885751488	F8747	0.871359121	0.876525702
F8315	1.000000000	1.000000000	F8749	0.903536034	0.907872402
F8332	0.926781284	0.961189022	F8750	0.850731578	0.873729751
F8340	0.945767366	0.963557691	F8752	1.000000000	1.000000000
F8350	0.972264789	0.978330323	F8783	0.796160736	0.910622972
F8353	0.972468091	1.000000000	F8786	1.000000000	1.000000000
F8357	0.80975047	0.810756307	F8787	1.000000000	1.000000000
F8359	0.880764729	0.931753971	F8794	0.950756285	0.979774636
F8377	0.934278599	0.94863399			

TABLE A.7: Efficiency scores of stores of store format “F” under CRS and VRS (continued).

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
G0141	0.781101224	0.788902755	G0592	0.899393217	0.929674317
G0155	1.000000000	1.000000000	G0603	0.899318695	0.918695523
G0164	1.000000000	1.000000000	G0616	0.997346082	0.998198871
G0169	0.873217016	0.934043538	G0629	0.882758199	0.887991441
G0181	0.905232484	0.906048589	G0643	0.872983831	0.875482717
G0184	0.9363583	0.953237937	G0662	0.978663935	0.984549797
G0188	0.859662194	0.885276012	G0666	0.896430499	0.896766143
G0190	0.931207942	0.985586531	G0674	0.857475168	0.88328745
G0192	0.908147421	0.92923415	G0704	0.970932904	0.979791068
G0194	0.951696349	0.970980103	G0721	0.898265668	0.905311035
G0203	0.923519083	0.923556489	G0748	0.867307063	0.882740881
G0205	0.954124755	0.973240547	G0779	0.910027132	0.943121574
G0206	0.875082654	0.896665671	G0780	0.798886807	0.849976042
G0208	0.863090417	0.899432834	G0797	1.000000000	1.000000000
G0211	0.976004288	1.000000000	G0935	0.885541699	0.896022192
G0218	0.916280649	0.951095124	G0971	0.91262577	0.91914204
G0220	0.868026842	0.89836204	G0975	0.941857571	0.942091724
G0221	0.896448457	0.917241933	G0981	0.936096409	1.000000000
G0227	1.000000000	1.000000000	G0989	0.898594032	0.96750009
G0235	1.000000000	1.000000000	G0996	0.984540547	1.000000000
G0245	0.777778981	0.880991926	G4172	0.960236025	0.96194261
G0255	0.941025576	0.942240267	G4209	0.958733183	0.967447793
G0256	0.915028462	0.916794783	G4244	1.000000000	1.000000000
G0270	0.97333792	0.985958294	G4256	0.995023032	1.000000000
G0271	0.97792597	0.987699968	G4266	0.949298321	0.95932518
G0272	0.836143552	0.889070609	G4278	0.908132302	0.917789688
G0297	0.86040719	0.8851632	G4527	0.822036437	0.85315975
G0304	0.942806905	0.95084735	G6120	0.780424066	0.86169607
G0309	0.989993664	1.000000000	G6128	1.000000000	1.000000000
G0318	0.882823706	0.891945012	G6132	0.862592937	0.897957033
G0324	1.000000000	1.000000000	G6133	0.914067049	0.950927048
G0326	0.923864521	0.924123714	G6139	0.982205221	0.984339578
G0336	0.971963202	0.997875105	G6140	0.885075449	0.90327267
G0361	0.901569193	0.942395935	G6141	0.960913287	1.000000000
G0369	0.942749626	0.971430943	G6143	0.759605837	0.773961834
G0372	0.957477141	0.957515623	G6158	0.878533528	0.885842279
G0373	1.000000000	1.000000000	G6160	0.868170871	0.903242178
G0382	0.961709859	0.999993677	G6162	0.88800095	0.944710208
G0396	0.827532874	0.862278722	G6163	0.841260981	0.86428857
G0413	0.908184886	0.918666118	G6170	0.95996547	0.960748334
G0428	0.974456	0.992556089	G6171	0.912447543	0.934278552
G0432	0.897211506	0.916070169	G6173	0.924916305	0.926342652
G0440	0.991731707	1.000000000	G6174	0.980916075	0.991337508
G0442	0.806354923	0.870331274	G6175	1.000000000	1.000000000
G0445	0.771936539	0.835168949	G6184	0.892870491	0.92680958
G0479	0.811668159	0.874887067	G6188	0.834425348	0.935745777
G0483	0.867561402	0.894158885	G6194	0.931283623	0.935724014
G0489	0.876992655	0.877512037	G6200	0.920091704	0.923410853
G0501	0.855399177	0.871109004	G6211	0.84773856	0.88029778
G0511	0.96782802	0.97342271	G6213	0.965115578	0.970857963
G0512	1.000000000	1.000000000	G6214	0.897143324	0.920724382
G0540	0.887360552	0.918736165	G6216	0.829910452	0.879980614
G0565	0.944273686	0.976871323	G6218	0.931530298	0.951758922
G0572	0.914451891	0.915991673	G6220	0.957462308	0.983505514
G0580	1.000000000	1.000000000	G6221	0.905693255	0.937536029

TABLE A.8: Efficiency scores of stores of store format “G” under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
G6222	0.988129343	0.995683613	G6376	0.96173885	0.962671169
G6225	0.978833905	1.000000000	G6377	1.000000000	1.000000000
G6232	0.975545681	1.000000000	G6379	0.887494144	0.89351912
G6234	0.958892343	0.974573037	G6381	0.971125493	0.984943171
G6236	0.952636276	0.959629049	G6382	0.85207995	0.879323163
G6240	0.952056842	0.958809259	G6384	0.943668508	0.947727968
G6243	0.942459641	0.96654169	G6385	0.928142281	0.931246215
G6244	0.949729274	0.961620542	G6397	0.855275996	0.919638738
G6245	0.877299135	0.906375351	G6399	1.000000000	1.000000000
G6246	0.823772917	0.845861592	G6401	0.963267737	0.981588805
G6252	0.878826464	0.928242083	G6413	0.817924644	0.853878916
G6266	0.931614428	0.976657015	G6418	0.935889643	0.979365762
G6268	0.970300334	0.978948605	G6419	0.875028252	0.879082574
G6269	0.923959065	0.928792717	G6420	0.945346502	0.951484157
G6273	0.902342873	0.902711461	G6421	0.949218876	0.949303865
G6274	0.8825569	0.893735417	G6422	0.859415528	0.884055249
G6285	0.863893697	0.868170815	G6430	0.890759251	0.90682382
G6287	1.000000000	1.000000000	G6432	0.948540449	1.000000000
G6294	0.970078956	1.000000000	G6434	0.802136478	0.836371659
G6295	0.904658713	0.905468342	G6435	0.970166213	0.989665307
G6299	0.927163461	0.942920427	G6437	0.886049081	0.911381616
G6301	0.960847348	0.97081893	G6439	0.883874002	0.888790331
G6302	0.9960542	0.996091823	G6441	0.879159667	0.927829471
G6303	0.944431641	0.948300595	G6443	0.859076998	0.908708036
G6308	0.915253567	0.933813734	G6445	0.858605318	0.904646422
G6311	0.762299129	0.814796515	G6448	1.000000000	1.000000000
G6314	0.884858479	1.000000000	G6452	0.920468862	0.938791894
G6317	1.000000000	1.000000000	G6456	0.924144688	0.991344458
G6325	0.896471119	0.905683286	G6457	0.933908712	0.961969277
G6326	0.847936049	0.872579865	G6458	1.000000000	1.000000000
G6327	0.855597625	0.877283313	G6460	0.907960672	0.9134173
G6328	0.825458592	0.845765738	G6461	0.967935638	0.98171309
G6330	1.000000000	1.000000000	G6462	0.866570532	0.901340936
G6331	1.000000000	1.000000000	G6464	0.948331878	0.953076373
G6332	0.917964685	0.946343473	G6467	0.967365388	0.970721314
G6334	0.930492771	0.987067293	G6468	0.974838049	0.989450579
G6335	0.885560548	0.915637753	G6469	1.000000000	1.000000000
G6336	0.965923905	0.979599837	G6470	1.000000000	1.000000000
G6337	0.907007699	0.937207729	G6471	0.866412591	0.876089769
G6338	0.95680708	0.968111371	G6481	0.937908683	0.961695881
G6339	0.987299425	1.000000000	G6482	0.869534343	0.903966171
G6342	0.968235277	0.970071357	G6484	0.943791326	0.945805825
G6345	0.85374545	0.909641391	G6492	0.962225176	0.986541585
G6346	0.888787148	0.921215274	G6493	0.926787988	0.926788581
G6350	0.949664411	0.950005884	G6496	0.912425984	0.924771651
G6351	0.97167549	1.000000000	G6500	1.000000000	1.000000000
G6352	0.942983946	0.945519839	G6560	0.906424322	0.950322696
G6353	0.890321551	0.94452974	G6577	0.813463506	0.856472248
G6361	0.867468658	0.881023837	G8031	0.877719711	0.898622189
G6363	1.000000000	1.000000000	G8032	1.000000000	1.000000000
G6365	0.842806954	0.853303518	G8063	1.000000000	1.000000000
G6367	0.943980697	0.954621912	G8081	0.863140917	0.893975124
G6368	0.987594651	0.98762758	G8094	0.874433158	0.898182528
G6372	0.900499538	0.917215355	G8095	1.000000000	1.000000000
G6375	1.000000000	1.000000000	G8104	0.919625416	0.933004732

TABLE A.9: Efficiency scores of stores of store format “G” under CRS and VRS (continued).

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
G8107	0.952946986	0.972790939	G8549	0.957577867	0.969282513
G8123	0.985863944	0.985908451	G8552	0.942703599	0.945046267
G8213	0.918452667	0.921807856	G8574	0.945628423	0.945716319
G8217	0.950486951	0.998344606	G8587	0.898943147	0.903763103
G8262	0.816052399	0.86786427	G8588	0.911038798	0.911124671
G8266	0.902870947	0.923272299	G8590	0.952985145	0.980058635
G8269	0.92655353	0.986434664	G8592	0.950988035	0.960444099
F8279	0.865215796	0.873246333	G8611	0.878462129	0.92986628
G8285	0.970994756	0.984102713	G8656	0.885911804	0.904398228
G8297	0.904531319	0.923482408	G8663	0.881322356	0.89603555
G8306	0.911278109	0.943649966	G8664	0.96270046	0.987714172
G8326	0.911178242	0.95209292	G8668	0.964625287	0.985535026
G8349	1.000000000	1.000000000	G8671	0.883095213	0.937869177
G8354	0.894505246	0.905680038	G8679	0.922400726	0.985119757
G8375	0.883276906	0.891274731	G8683	0.904054557	0.911137899
G8403	0.952552192	0.959363315	G8689	0.885301748	0.932878319
G8409	0.865352548	0.869801139	G8690	0.952606645	0.971494512
G8419	0.906013748	1.000000000	G8694	0.948225327	0.960740246
G8424	1.000000000	1.000000000	G8698	0.915094835	0.945718882
G8437	0.862349468	0.903037963	G8703	0.868603291	0.869053925
G8438	1.000000000	1.000000000	G8711	0.916938844	0.950276403
G8447	0.857025948	0.968562268	G8722	0.855934693	0.961841098
G8454	0.892317977	0.893438419	G8738	0.85115884	0.861648031
G8463	0.893697418	0.954473241	G8741	0.872914318	0.913581837
G8467	0.92859676	0.963764339	G8744	0.908283945	0.908959339
G8490	0.989986959	0.992025283	G8746	0.916314165	0.964900954
G8494	0.812658973	0.836933991	G8755	0.918208406	0.931353006
G8499	1.000000000	1.000000000	G8759	0.914749946	0.917080954
G8500	0.899289694	0.927564297	G8766	0.926535522	0.93483319
G8502	0.864828574	0.907994625	G8772	0.906207386	0.907355059
G8512	0.977790881	1.000000000	G8778	0.975804168	1.000000000
G8513	0.905697998	0.924418714	G8784	0.906843509	0.956703855
G8535	0.946264411	0.971831428	G8795	0.849280669	0.860219187
G8548	0.905558133	0.946243229			

TABLE A.10: Efficiency scores of stores of store format “G” under CRS and VRS (further continued).

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
H0109	0.79502792	0.895082989	H0945	0.911703596	0.912058464
H0119	0.95382753	0.968824194	H0949	0.946741395	0.951067787
H0136	0.935579317	0.937136617	H0958	0.873171397	0.949944915
H0139	0.853514987	0.853613238	H0960	0.889710321	0.947850217
H0146	0.96744518	1.000000000	H0964	0.915503866	0.915960101
H0172	0.97217321	0.97217321	H0967	0.92932959	0.954411895
H0179	0.820243154	0.820243154	H0984	0.795451974	0.817160189
H0185	0.916616169	0.918892008	H0995	0.968984485	0.976493219
H0187	0.920639171	0.934851363	H4101	0.871260897	0.924689255
H0197	0.953131546	0.953369813	H4196	0.870218244	0.912605631
H0198	0.846761362	0.850835273	H4226	0.8922553	0.8922553
H0223	0.834931193	0.838090228	H4230	0.836003281	0.837243134
H0224	0.8711732	0.917559818	H4255	0.981024932	0.997102293
H0257	0.900357141	0.902818674	H4257	0.92145705	0.942041392
H0279	0.867755053	0.892874432	H4280	0.79987419	0.816687732
H0289	0.911846245	0.948858385	H4506	0.928051994	0.931195774
H0291	0.900373768	0.908446058	H4525	0.951883251	0.953119902
H0307	0.922788487	0.92639348	H4530	0.92491963	0.92881358
H0311	0.927092897	0.935686078	H6101	0.881599512	0.883332425
H0314	0.930342204	0.930342204	H6105	0.964666337	0.967994409
H0320	0.886624544	0.887598486	H6118	0.922771763	0.924334284
H0338	1.000000000	1.000000000	H6123	0.889777457	0.923024305
H0340	0.864596095	0.913122104	H6126	0.979695031	1.000000000
H0392	0.865527727	0.876095813	H6144	0.966786493	0.9672047
H0397	0.853024931	0.853066857	H6167	0.889834699	0.921691599
H0434	0.960215409	0.960215409	H6176	0.685238838	0.713508268
H0456	1.000000000	1.000000000	H6177	0.872107763	0.936490126
H0461	0.887606217	0.88761177	H6178	0.901890768	0.932490993
H0465	0.846238411	0.846698554	H6180	0.950810516	0.958157826
H0536	1.000000000	1.000000000	H6193	0.930101644	0.930648544
H0551	1.000000000	1.000000000	H6202	0.86653675	0.867035978
H0554	1.000000000	1.000000000	H6205	0.961150794	0.963040637
H0555	0.913325964	0.96996345	H6208	0.924004615	0.936076399
H0566	0.843542123	0.845187329	H6215	0.893505687	0.936099004
H0575	0.959882163	0.969103877	H6217	0.832000938	0.842120801
H0588	0.833401693	0.837898662	H6219	0.939943012	0.951776778
H0597	0.959914105	0.962398015	H6226	0.831989313	0.868721608
H0613	0.847915717	0.860800289	H6227	0.909958247	0.952264778
H0621	0.930024017	0.930065866	H6229	0.955172592	0.955172592
H0638	0.936607391	0.94746202	H6230	0.808931782	0.837775936
H0663	0.971236782	0.971236782	H6231	0.981349799	0.988179643
H0688	0.935961406	0.936693015	H6248	0.803032761	0.850947411
H0712	0.751817356	0.763062021	H6264	0.924454295	0.924454295
H0724	0.879248976	0.919226022	H6265	0.934197233	0.938284936
H0740	0.893831516	0.951285397	H6272	0.950374405	0.989786066
H0742	0.937414965	0.980560759	H6280	0.809095965	0.818743286
H0743	0.843572457	0.844177927	H6320	0.936531196	0.938597501
H0772	0.808277118	0.860568767	H6321	0.735965325	0.74336716
H0783	0.876816268	0.911063654	H6322	0.750744482	0.808832325
H0785	0.861947224	0.911562726	H6323	0.883340684	0.910495228
H0796	0.973775471	0.973775471	H6341	0.936419145	0.9369974
H0923	0.966900334	1.000000000	H6344	0.991578394	0.992633066
H0925	0.90850301	0.950880035	H6349	0.832353076	0.888299583
H0926	1.000000000	1.000000000	H6355	1.000000000	1.000000000
H0941	0.916734277	0.918128271	H6357	0.889514993	0.895573121

TABLE A.11: Efficiency scores of stores of store format “H” under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
H6360	0.869528513	0.882300455	H6530	0.71282307	0.71282307
H6364	0.869528513	0.882300455	H6532	0.964464212	0.989955711
H6366	1.000000000	1.000000000	H6535	0.886885031	0.9641503
H6369	1.000000000	1.000000000	H6536	0.848962975	0.849515396
H6371	0.872703295	0.896982191	H6539	0.703392784	0.72952784
H6386	0.852945762	0.857787496	H6541	0.933868821	0.956785172
H6387	0.932387049	0.963187224	H6544	0.980407912	1.000000000
H6389	0.918888524	0.929070102	H6546	0.908644933	0.921092315
H6395	0.916952374	0.919327591	H6547	0.879444632	0.902243553
H6396	0.929604249	0.929604249	H6548	0.899399075	0.934113856
H6400	0.964633174	0.970716048	H6551	0.97638151	0.97638151
H6403	0.872784267	0.888473621	H6552	0.886402065	0.886402065
H6405	0.841369634	0.869977264	H6553	0.773397307	0.774111217
H6406	0.76430724	0.793367655	H6554	0.935171795	0.935171795
H6407	0.865823715	0.865823715	H6555	0.857333353	0.857333353
H6408	0.948986128	0.949343312	H6564	0.880805053	0.925178941
H6409	0.824791439	0.827017469	H6570	0.968537523	0.970229188
H6411	0.943671032	0.948335118	H6571	0.860321333	0.861806158
H6414	0.902166988	0.930858694	H6572	0.92519849	0.925210809
H6416	0.904153407	0.916148499	H6573	0.903511422	0.918727378
H6423	0.876163645	0.933972025	H6575	0.808226906	0.879867832
H6424	0.92823793	0.968534491	H6584	0.783202274	0.790672866
H6426	1.000000000	1.000000000	H6588	0.854865758	0.863901992
H6428	0.931493325	0.93286187	H6590	0.870465025	0.877990312
H6431	0.879329567	0.879329567	H6600	0.86073456	0.860919253
H6436	0.830139943	0.846711532	H6601	0.988780914	1.000000000
H6440	0.896867162	0.903529386	H6607	0.980265187	1.000000000
H6442	0.836202769	0.857276891	H6608	0.905050903	0.914000862
H6444	0.898693634	0.919596799	H6610	0.736668506	0.773010716
H6447	0.957078803	0.957078803	H6612	0.971095435	1.000000000
H6449	0.888593028	0.893305646	H6617	0.873572508	0.87992648
H6450	0.893216655	0.903447008	H6620	0.859052031	0.859307969
H6451	0.863215007	0.877642262	H6622	0.773815269	0.833826311
H6455	0.900029993	0.928865892	H6625	0.97252067	0.972771773
H6463	0.938967223	0.957140589	H6627	1.000000000	1.000000000
H6472	0.973811278	1.000000000	H6630	0.874282605	0.894200838
H6473	0.633141938	0.743163473	H6633	0.909242635	0.909242635
H6478	0.783866767	0.868696607	H6634	0.992629214	0.993248103
H6483	0.942108689	0.942108689	H6635	0.940279271	0.940447667
H6495	0.896888637	0.931309239	H6639	1.000000000	1.000000000
H6498	0.816471507	0.823815504	H6640	1.000000000	1.000000000
H6499	0.937677409	0.939207842	H6641	0.954657471	0.954657471
H6501	0.848062446	0.864392307	H6642	0.935123697	0.935985212
H6503	0.837646498	0.837909235	H6643	1.000000000	1.000000000
H6504	0.89707625	0.930015405	H6645	0.866643673	0.866643673
H6508	0.96457247	0.965585938	H6647	0.895994774	0.916556745
H6509	0.93146511	0.93146511	H6650	0.895994774	0.916556745
H6514	0.841907657	0.863804946	H6652	0.884538188	0.907457235
H6516	0.823306972	0.842041554	H6653	0.966288237	0.96655064
H6517	0.871394046	0.895407465	H6654	0.941779945	0.98344691
H6519	0.858425674	0.904997595	H6655	0.780838458	0.800173157
H6521	0.874973862	0.925672566	H6656	0.820323717	0.83509386
H6523	0.968516266	0.977123019	H6659	0.761145096	0.761145096
H6524	0.935884117	0.937555585	H6660	0.928201489	0.934135196
H6525	0.871975632	0.873712631	H6661	1.000000000	1.000000000

TABLE A.12: Efficiency scores of stores of store format “H” under CRS and VRS (continued).

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
H6664	1.000000000	1.000000000	H8393	0.89997954	0.963747683
H6665	1.000000000	1.000000000	H8396	0.958640768	0.958640768
H6667	0.903896913	0.926835446	H8397	1.000000000	1.000000000
H6669	0.983050823	0.991262603	H8400	0.943943535	0.946519385
H6671	0.922875667	1.000000000	H8401	0.886844885	0.888004202
H6672	0.920636919	0.922756882	H8411	0.811779429	0.814611048
H6673	0.953257744	0.991208722	H8412	0.95985625	0.973353511
H6676	1.000000000	1.000000000	H8414	1.000000000	1.000000000
H6679	0.980636802	0.98484524	H8415	0.843156598	0.843238344
H6680	0.80099003	0.808296131	H8417	0.940393785	0.941390011
H6682	0.84293759	0.937407601	H8432	0.360240192	0.378433518
H6683	1.000000000	1.000000000	H8449	0.893191008	0.894298243
H6684	0.917021206	0.950747302	H8450	0.920987701	0.921004281
H6686	0.961125218	0.971834041	H8461	0.958071222	0.969297677
H6687	0.971253611	1.000000000	H8477	0.963359827	0.968771105
H6688	1.000000000	1.000000000	H8487	0.902767178	0.902843738
H6689	0.987110938	0.987300363	H8501	0.937693212	0.937719924
H6690	1.000000000	1.000000000	H8514	0.855588364	0.855588364
H6692	1.000000000	1.000000000	H8515	0.943280456	0.945911317
H6693	0.86381624	0.964345688	H8516	0.865124808	0.894664019
H6698	1.000000000	1.000000000	H8522	0.980038484	0.980038484
H6700	0.837739319	0.991999356	H8525	0.923899594	0.925576597
H6704	0.526362101	0.546584724	H8538	0.950605132	0.950731201
H6713	0.734074868	0.740840137	H8539	0.991401937	0.991975373
H8013	0.900204621	0.904738849	H8541	0.932111108	0.954420673
H8026	0.832024507	0.865817874	H8545	0.889688399	0.903619238
H8055	0.912260363	0.952579064	H8551	0.872478545	0.885087087
H8086	0.896074196	0.896074196	H8569	0.950660964	0.953380739
H8087	0.828978285	0.829338599	H8571	0.850686033	0.850686033
H8097	0.903965102	0.980167668	H8573	0.917945376	0.948824558
H8101	0.93899654	0.955645729	H8586	0.886227244	0.947326132
H8103	0.895497199	0.895497199	H8593	0.910145649	0.945892956
H8105	0.959798371	0.959798371	H8648	0.976042778	0.976042778
H8109	0.973547153	0.973887885	H8654	0.920377757	0.920377757
H8133	0.869029704	0.87757253	H8655	0.968705351	0.983257334
H8134	0.913041697	0.92018547	H8666	0.845470838	0.84787317
H8211	0.899613865	0.943375289	H8682	1.000000000	1.000000000
H8218	0.807785023	0.865211833	H8685	0.817494061	0.832162088
H8227	1.000000000	1.000000000	H8688	0.912767709	0.959741921
H8235	0.820634425	0.860008602	H8692	0.879735692	0.879735692
H8258	0.904003312	0.917122742	H8697	0.809521662	0.895991343
H8271	0.89261601	0.897821089	H8706	1.000000000	1.000000000
H8274	0.927084228	0.949957176	H8715	0.922072601	0.932779048
H8278	0.927092798	0.932043884	H8729	0.908861269	0.940495398
H8311	0.890282277	0.896153513	H8730	0.947115387	0.947402259
H8317	0.904246517	0.967018681	H8734	0.976494583	0.976494583
H8321	0.922127353	1.000000000	H8737	0.975621437	0.982070246
H8323	1.000000000	1.000000000	H8756	0.917446499	0.920448307
H8336	0.876280382	0.9366602	H8760	0.877725595	0.877725595
H8337	0.779168095	0.779932264	H8773	0.979971557	0.979971557
H8346	0.857028638	0.941147903	H8776	0.874965243	0.908156784
H8352	0.93266057	0.936957712	H8779	0.904048559	0.936663928
H8367	0.884853917	0.895837832	H8780	0.839329553	0.839329553
H8373	0.863457619	0.865775749	H8782	0.845255881	0.873901773
H8374	0.968903916	0.968903916	H8792	1.000000000	1.000000000
H8378	0.880493467	0.881301859	H8793	0.909483473	0.957077623
H8380	0.938728115	0.938728115	H8797	0.929143233	0.929597385

TABLE A.13: Efficiency scores of stores of store format “H” under CRS and VRS (further continued).

APPENDIX B

Efficiency scores per region

Appendix B is a compilation of the efficiency scores of stores per region. The scores may differ when the calculation group considers all stores as decision-making units. For comparability, the calculation groups are divided into regions and the season of products considered. The unique store ID number is provided, along with the efficiency score under constant and variable returns to scale, respectively.

Store ID	θ_{CRS}	θ_{VRS}
W170105	1.000000000	1.000000000
W170139	0.892199213	0.923418313
W170197	0.88949049	0.922804234
W170198	0.947756947	0.952747133
W170202	0.976489058	0.992980034
W170231	0.970171718	0.975357789
W170328	0.931596473	0.959425673
W170448	1.000000000	1.000000000
W170478	0.908271358	0.952273142
W170734	0.956514303	0.98074742
W170982	1.000000000	1.000000000
W174168	0.902940879	0.932346806
W174172	1.000000000	1.000000000
W174203	0.818329071	0.899759486
W175510	0.818329071	0.899759486
W175612	0.818329071	0.899759486
W175617	0.818329071	0.899759486
W175652	0.818329071	0.899759486
W176201	1.000000000	1.000000000
W176286	0.946014205	0.962135017
W176324	1.000000000	1.000000000

Store ID	θ_{CRS}	θ_{VRS}
W176444	0.952615032	0.965983397
W176485	1.000000000	1.000000000
W176487	0.978983382	0.998005809
W176516	0.786861739	0.813758049
W176541	1.000000000	1.000000000
W176554	0.96472961	0.977963605
W176584	0.928366288	0.948917865
W176638	0.883253273	0.893072719
W176649	0.917382648	0.918489673
W176666	1.000000000	1.000000000
W176674	1.000000000	1.000000000
W176679	1.000000000	1.000000000
W176709	1.000000000	1.000000000
W178263	1.000000000	1.000000000
W178510	1.000000000	1.000000000
W178515	0.950013992	0.962497376
W178531	1.000000000	1.000000000
W178683	0.966326556	0.983579744
W178732	0.950799512	0.976112679
W178752	0.881480448	0.925963301

TABLE B.1: Efficiency scores of stores in the Southern Namibia region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}
W170167	0.999973592	1.000000000
W170219	0.953083539	0.977088106
W170256	0.925197193	0.938005154
W170297	0.87891349	0.900087863
W170323	1.000000000	1.000000000
W170326	0.889087628	0.919417556
W170432	0.956978819	0.957966449
W170447	0.928135186	0.932452348
W170467	0.797081348	0.829660314
W170507	0.933460341	1.000000000
W170981	0.910852528	0.920550725
W170984	0.89216609	0.910497772
W175586	0.89216609	0.910497772
W175627	0.89216609	0.910497772
W176144	0.937402977	0.959902527
W176174	0.999817699	1.000000000
W176229	0.969234394	0.996464834
W176469	0.95901172	0.979080321
W176506	0.952003309	0.95205717
W176519	0.895197781	0.932052008
W176543	1.000000000	1.000000000
W176561	0.93420987	0.957122693
W176571	0.857676603	0.866899831
W176582	0.99905992	0.999060501
W176600	0.862005273	0.873399281
W176620	0.881535767	0.928179152

Store ID	θ_{CRS}	θ_{VRS}
W176623	0.914272313	0.914427245
W176624	0.920024847	0.920035397
W176625	0.957941488	0.97300812
W176626	0.777677065	0.820917502
W176637	0.929891006	0.934448766
W176656	0.971346378	1.000000000
W176669	0.941635167	0.965139376
W176688	1.000000000	1.000000000
W176708	1.000000000	1.000000000
W178271	0.961635415	0.992001572
W178272	1.000000000	1.000000000
W178298	0.925594164	0.956564143
W178340	0.938410109	0.958274105
W178370	1.000000000	1.000000000
W178382	0.810930377	0.857568045
W178391	1.000000000	1.000000000
W178393	0.977607545	1.000000000
W178412	0.978874776	1.000000000
W178420	0.860074138	0.872289037
W178493	1.000000000	1.000000000
W178499	1.000000000	1.000000000
W178520	0.922674435	0.94919493
W178521	0.821969426	0.837911942
W178529	0.969272495	0.974509477
W178539	0.940572963	0.950637678
W178546	0.904110327	0.928662862

TABLE B.2: Efficiency scores of stores in the Northern Namibia region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}
W170115	0.941426437	0.954039341
W170199	1.000000000	1.000000000
W170428	1.000000000	1.000000000
W170520	0.953602081	0.958568373
W170536	1.000000000	1.000000000
W170597	1.000000000	1.000000000
W170634	1.000000000	1.000000000
W170907	0.965637937	0.976815111
W170964	0.933387791	0.937029382
W170978	0.882127037	0.96517444

Store ID	θ_{CRS}	θ_{VRS}
W174530	1.000000000	1.000000000
W176193	0.95698181	0.957373533
W176532	1.000000000	1.000000000
W176713	0.772872586	1.000000000
W178089	0.963965532	1.000000000
W178292	0.96305984	0.987580324
W178323	1.000000000	1.000000000
W178346	0.932620191	1.000000000
W178357	0.877583939	0.8881949
W178512	1.000000000	1.000000000

TABLE B.3: Efficiency scores of stores in the Swaziland region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W170257	0.886131223	0.895901701	W178274	0.937666224	0.9528165
W170309	1.000000000	1.000000000	W178284	0.862244719	0.862398642
W170402	0.917327207	0.977652765	W178310	0.906077051	0.906365913
W170549	0.942296543	0.944956292	W178311	0.874065432	0.87757929
W170551	1.000000000	1.000000000	W178325	0.93765387	0.938302403
W170552	0.999420613	1.000000000	W178348	0.924628932	0.926932628
W170554	1.000000000	1.000000000	W178349	0.954750821	0.956140539
W170555	0.888861909	0.889319898	W178350	0.972077763	0.972700921
W170556	0.936220278	0.942480117	W178373	0.801382856	0.801865645
W170575	0.944702438	0.985486661	W178374	0.884831232	0.899921351
W170905	1.000000000	1.000000000	W178396	0.920158388	0.920297019
W170926	1.000000000	1.000000000	W178397	1.000000000	1.000000000
W170945	0.939940395	0.941877975	W178398	0.910653945	0.953451686
W176105	0.930101815	0.932551494	W178399	0.893213654	0.908920281
W176119	0.926229921	0.991426054	W178414	0.986717563	1.000000000
W176135	0.945054002	0.993560595	W178415	0.857667492	0.859863131
W176208	0.887902382	0.896981401	W178476	0.994262481	0.995294991
W176234	0.944607197	0.961621174	W178501	0.952693604	0.953171748
W176264	0.955059278	0.955817229	W178537	0.972573538	0.972598886
W176295	0.874568473	0.883494145	W178538	0.990371657	0.997864119
W176341	0.941940909	0.942491171	W178569	0.891353799	0.895086807
W176361	0.786920547	0.823043138	W178571	0.894517	0.896717461
W176435	0.914940102	0.94072196	W178593	0.853429431	0.87341715
W176440	1.000000000	1.000000000	W178600	1.000000000	1.000000000
W176447	0.943895762	0.944555985	W178665	0.84560517	0.85204507
W176456	0.902893125	0.915928876	W178706	1.000000000	1.000000000
W176470	0.985800838	0.994853316	W178730	0.929276502	0.934494799
W176525	0.855454548	0.899600776	W178731	0.834520091	0.850643422
W176551	0.973259626	0.973929775	W178734	1.000000000	1.000000000
W176560	0.915013298	0.915049482	W178747	1.000000000	1.000000000
W178087	0.873387143	0.873484884	W178749	0.956716165	0.957195882
W178204	0.922728537	1.000000000	W178750	0.947084309	1.000000000
W178226	0.930313898	0.930324446	W178786	0.959470904	0.988388034
W178227	1.000000000	1.000000000	W178797	0.937597233	0.977404813

TABLE B.4: Efficiency scores of stores in the Botswana region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W170104	0.961472875	0.968792008	W170730	1.000000000	1.000000000
W170109	0.931230228	0.962085105	W170732	0.842796673	0.866770722
W170111	0.903147411	0.937519036	W170780	0.689993042	0.749167844
W170144	1.000000000	1.000000000	W170789	0.923407722	0.940059993
W170148	0.89617145	0.923632382	W170970	0.868372633	0.918579305
W170151	0.850061569	0.921370869	W174101	0.8830909	0.901419929
W170154	0.892946729	1.000000000	W174244	0.932945589	0.966933449
W170156	0.89255945	0.918286831	W176136	1.000000000	1.000000000
W170173	0.881545689	0.948160667	W176146	0.9162631	0.97435477
W170184	0.938145116	0.958918471	W176161	0.965829564	0.980532757
W170188	0.785208211	0.868392191	W176170	1.000000000	1.000000000
W170190	0.97158134	0.99200093	W176268	1.000000000	1.000000000
W170206	0.874432797	0.923490565	W176277	0.961705209	1.000000000
W170226	0.907387486	0.914276234	W176279	0.818736685	0.83594144
W170228	0.836400841	0.863321285	W176317	0.827416494	0.849053401
W170234	0.927847241	0.946799739	W176349	1.000000000	1.000000000
W170247	0.881491884	0.882288509	W176351	1.000000000	1.000000000
W170248	1.000000000	1.000000000	W176355	0.954800699	0.96280485
W170251	1.000000000	1.000000000	W176414	0.938082646	0.994908339
W170270	0.915013925	0.9666114	W176455	0.851260781	0.872904335
W170290	0.898535233	1.000000000	W176465	0.847849008	0.888359457
W170333	0.910715799	0.927423476	W176534	0.997191563	1.000000000
W170341	0.811969781	0.831023415	W176575	0.910568754	0.91418817
W170344	0.879058517	0.884677463	W176590	0.842618103	0.902683631
W170361	0.851018657	0.882567979	W176643	1.000000000	1.000000000
W170366	1.000000000	1.000000000	W176670	1.000000000	1.000000000
W170369	0.931270566	0.95658081	W176671	1.000000000	1.000000000
W170372	0.925587319	0.948957267	W178018	0.904949892	0.928943438
W170382	0.953058296	1.000000000	W178042	0.695281769	1.000000000
W170386	0.933321463	0.976323117	W178046	0.837433577	0.874355908
W170391	0.913344926	0.995789248	W178092	0.84205767	0.852483353
W170405	0.978696838	0.998658593	W178133	0.978671887	0.982897633
W170411	0.873159958	0.881385177	W178365	0.883204649	0.904155895
W170417	1.000000000	1.000000000	W178388	0.898691321	0.900246056
W170457	0.912486974	0.93994614	W178419	0.880217207	0.940891621
W170494	0.838630459	0.843902657	W178487	0.916916917	0.917284184
W170505	0.862687256	0.903815185	W178674	0.974962752	1.000000000
W170565	0.916403106	0.984793999	W178679	0.949238252	0.956817954
W170570	0.855054012	0.92819375	W178696	1.000000000	1.000000000
W170621	0.915254761	0.922733524	W178726	0.911492279	0.960319034
W170723	0.818591692	0.874351039	W178735	0.901711137	0.970276614
W170724	0.874890748	0.936056541	W178792	1.000000000	1.000000000
W170727	0.860812385	0.918529777			

TABLE B.5: Efficiency scores of stores in the Cederberg region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W170221	0.950102898	0.95776469	W176432	0.998063661	0.998278311
W170255	0.969987398	0.982108933	W176460	0.945459203	0.953472524
W170271	1.000000000	1.000000000	W176461	0.9742136	0.986355027
W170291	0.88955877	0.940956705	W176480	0.916224294	0.944228753
W170303	0.974241488	0.977422088	W176490	0.931202607	0.956222843
W170335	1.000000000	1.000000000	W176495	0.997393056	0.998562139
W170430	0.980101615	1.000000000	W176512	0.978061432	0.990190899
W170542	0.998494257	1.000000000	W176562	0.939274672	0.966017347
W170630	0.942206942	0.953428566	W176565	0.873467739	0.943571555
W170638	0.959005092	0.993640624	W176570	0.94007584	0.954217219
W170639	0.967987356	0.98719369	W176578	0.885634193	0.906618051
W170674	0.95155288	1.000000000	W176588	0.978963907	1.000000000
W170797	0.996961583	1.000000000	W176641	1.000000000	1.000000000
W170924	0.915121011	0.952413644	W176642	1.000000000	1.000000000
W170989	0.921430473	0.980157482	W176654	0.96293644	1.000000000
W174506	0.939828893	0.941863805	W176677	1.000000000	1.000000000
W176115	0.96557455	0.981071856	W178079	0.961267332	0.966280282
W176127	0.873701714	0.879496698	W178114	0.899141356	0.915158821
W176138	1.000000000	1.000000000	W178218	0.826848998	0.949088701
W176180	0.989696813	1.000000000	W178220	0.967289902	1.000000000
W176184	0.914140741	0.938598509	W178258	0.972873417	0.984046005
W176194	0.964301604	0.986834452	W178269	0.93030812	0.977928978
W176205	1.000000000	1.000000000	W178278	0.952524272	0.953052339
W176220	1.000000000	1.000000000	W178306	0.948203514	0.968718405
W176221	1.000000000	1.000000000	W178378	1.000000000	1.000000000
W176243	1.000000000	1.000000000	W178403	1.000000000	1.000000000
W176246	0.934622132	0.949110719	W178430	1.000000000	1.000000000
W176273	0.96200637	0.967480839	W178431	0.962777225	0.963389478
W176274	0.951420869	0.954688877	W178437	0.916223003	0.941158502
W176321	0.935060798	0.95361079	W178438	1.000000000	1.000000000
W176331	1.000000000	1.000000000	W178489	0.91187167	0.952164661
W176338	0.996664285	1.000000000	W178517	0.939516306	0.963170402
W176358	0.950409655	0.985186849	W178592	0.976590211	1.000000000
W176363	0.954852916	0.958787796	W178667	1.000000000	1.000000000
W176367	0.955969122	0.980690356	W178668	1.000000000	1.000000000
W176371	0.962572224	0.977247999	W178671	0.966971842	0.985351863
W176376	0.978213542	0.988461653	W178673	1.000000000	1.000000000
W176377	1.000000000	1.000000000	W178687	0.938762079	0.956481167
W176384	0.995790505	1.000000000	W178689	0.981273731	0.990809679
W176386	0.922491001	0.930364461	W178739	0.952319262	0.970093192
W176390	1.000000000	1.000000000	W178767	1.000000000	1.000000000
W176402	0.998216327	1.000000000	W178772	0.986478105	0.989383871
W176422	0.91036708	0.941951579			

TABLE B.6: Efficiency scores of stores in the Kwenya region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W170146	0.921867616	1.000000000	W176366	1.000000000	1.000000000
W170180	0.732881724	0.909608754	W176372	0.968541378	0.975261549
W170223	0.906869971	0.973766951	W176397	0.929698092	0.939848468
W170267	0.852586348	0.983548694	W176413	0.818700229	0.898129922
W170279	0.911824991	0.959988236	W176421	1.000000000	1.000000000
W170289	0.970705578	0.99846184	W176428	0.941168955	1.000000000
W170307	0.884027223	0.956417584	W176436	0.824759187	0.931051742
W170373	0.930565529	0.989729815	W176443	0.820113507	0.8962064
W170400	0.935695189	1.000000000	W176466	0.860441148	0.970047054
W170409	0.777702001	0.872583486	W176482	0.913578357	0.980459419
W170413	0.91650325	0.965588415	W176483	0.832107929	0.993244435
W170418	1.000000000	1.000000000	W176502	0.848095401	0.859481375
W170435	0.781560315	0.88725334	W176524	0.851041634	0.992051052
W170484	1.000000000	1.000000000	W176526	0.957822438	1.000000000
W170499	0.864093788	0.905282392	W176546	0.991842903	1.000000000
W170544	0.951453919	1.000000000	W176550	0.875568199	1.000000000
W170591	0.836670576	0.92159127	W176553	0.662304221	0.86502628
W170610	1.000000000	1.000000000	W176557	0.896263973	0.911828334
W170666	0.793030004	0.881872993	W176558	0.872076947	0.965657562
W170689	0.927663623	1.000000000	W176559	0.886207577	0.971099438
W170796	0.901703552	0.989603321	W176569	0.819746974	0.89647375
W170958	1.000000000	1.000000000	W176606	0.798928662	0.873767129
W170995	1.000000000	1.000000000	W176619	0.873696402	0.910373097
W174209	0.976039707	1.000000000	W176683	1.000000000	1.000000000
W174255	0.883023319	1.000000000	W178023	0.7059097	0.848277177
W174512	0.741824794	0.898327408	W178134	0.858867491	0.931069517
W176104	0.893835248	1.000000000	W178295	0.960804721	0.989464919
W176122	0.725415399	0.876857162	W178384	0.941145009	0.978822892
W176126	1.000000000	1.000000000	W178450	0.977543342	0.988539526
W176141	0.723416306	0.891675432	W178467	0.891768421	0.922867155
W176163	0.824104512	0.935159574	W178511	0.866153112	0.906962348
W176222	0.830594843	0.944909428	W178516	0.907527157	0.91226424
W176235	0.865240447	0.929209325	W178551	0.782020392	0.927987933
W176241	0.874241389	0.964493592	W178552	0.936394799	0.957553461
W176245	0.886193032	0.997424345	W178553	0.829502745	0.926805972
W176252	0.879845413	0.951422167	W178591	0.967479081	0.983445569
W176269	0.849605741	0.920801045	W178681	1.000000000	1.000000000
W176284	0.822712369	1.000000000	W178698	0.989606718	1.000000000
W176285	0.885087379	0.932220265	W178705	0.804347103	0.983389047
W176330	0.942872494	0.94445077	W178711	1.000000000	1.000000000
W176333	1.000000000	1.000000000	W178755	0.922700748	0.989622847
W176335	0.763580531	0.956666814	W178759	0.917478813	0.924861471
W176365	0.677682984	0.882104157	W178769	0.930825639	0.962420811

TABLE B.7: Efficiency scores of stores in the Emfuleni region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W170102	0.869449303	1.000000000	W170955	0.729213559	0.788534149
W170112	0.866401572	0.884724274	W174119	0.810477066	0.907766005
W170113	1.000000000	1.000000000	W174231	0.90941972	0.982163441
W170119	0.851495285	0.946387023	W176128	0.793991493	0.927609395
W170131	0.851445674	0.943619939	W176143	0.666483767	0.740114172
W170150	0.776097665	0.965166244	W176160	0.863863069	0.907004143
W170152	0.904392526	0.938811382	W176162	0.935742606	0.985171205
W170160	0.61608007	1.000000000	W176188	0.735071748	0.944628209
W170172	0.97144856	0.980145168	W176215	0.917707327	0.956212281
W170179	0.823223751	0.871916132	W176227	0.647680503	0.956122834
W170182	0.755861687	0.778630202	W176248	0.698246873	0.891620274
W170183	0.77520123	0.900715981	W176287	0.727338642	0.85128425
W170194	0.913330385	0.92684751	W176294	0.970674016	0.986602439
W170215	0.632162533	0.798050712	W176308	0.748471567	0.936094394
W170225	0.770684314	0.833278749	W176311	0.69187737	0.733384445
W170236	0.65141127	0.797581102	W176322	0.715948048	0.877057579
W170245	0.875894513	0.941968601	W176360	0.777516168	0.862392709
W170268	0.718470608	0.827074966	W176375	1.000000000	1.000000000
W170288	0.796593447	0.94818064	W176399	0.76876613	0.883448209
W170320	0.981010747	1.000000000	W176406	0.70629829	0.86513163
W170340	0.732400012	0.879256933	W176426	0.58726925	0.6650026
W170371	0.757400062	0.851230457	W176438	0.695922402	0.86081334
W170374	0.724794532	0.733763449	W176458	0.876381016	0.893459181
W170390	0.726587823	0.8081544	W176478	0.841823144	0.917087349
W170392	0.769113965	0.881358479	W176610	0.688764525	0.782034208
W170396	0.793227044	0.805452572	W176645	0.89875713	0.952299549
W170406	0.841651594	0.851370134	W176661	0.984144753	0.99193912
W170492	0.882855876	0.919062234	W178012	0.89644103	0.919296116
W170513	0.634644792	0.751488937	W178059	0.649146686	0.727763119
W170592	0.79403922	0.963092576	W178065	0.715467485	0.816701529
W170594	0.690495711	0.752803522	W178067	0.713157082	0.894061114
W170598	1.000000000	1.000000000	W178111	0.838137755	0.93657212
W170599	0.854552363	0.947913188	W178315	0.610487308	0.647281021
W170601	0.793222458	0.898029958	W178377	0.943443761	0.945403407
W170613	0.745313466	0.755945128	W178400	0.976330791	0.981438561
W170704	0.962777907	0.994046119	W178447	0.936423122	1.000000000
W170710	0.652979431	0.685988543	W178500	0.880954677	0.958350251
W170712	0.640326186	0.660399477	W178678	0.735875817	0.774454564
W170718	0.704524374	0.850801853	W178713	0.785677927	0.913019069
W170721	0.903091638	0.916816972	W178729	0.721495021	0.943571478
W170733	0.839911026	0.938564452	W178751	0.631999502	0.724547077
W170783	0.777249197	0.908674587	W178756	0.775193317	0.878395816
W170923	0.713229543	0.748424652	W178787	0.851945533	0.94388299

TABLE B.8: Efficiency scores of stores in the Langeberg region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W170176	0.978603487	0.995116917	W176350	0.98435979	0.989028775
W170187	0.905187642	0.934510719	W176396	1.000000000	1.000000000
W170189	0.865555537	0.890337222	W176408	0.914932101	0.929822606
W170191	0.886321679	0.988117331	W176418	0.973748764	0.993817081
W170192	0.881540143	0.91845349	W176419	0.876717065	0.913698584
W170203	0.946195628	0.958334755	W176492	0.995713393	1.000000000
W170205	0.95321238	0.958328538	W176496	0.878779843	0.934098374
W170237	0.93093249	0.931411193	W176505	0.803698305	0.920192176
W170282	1.000000000	1.000000000	W176509	0.95623816	0.969978172
W170283	0.882103135	0.905438624	W176542	0.948679902	0.986708384
W170336	1.000000000	1.000000000	W176563	0.891624014	0.906896252
W170339	1.000000000	1.000000000	W176591	0.91548634	0.920414441
W170350	0.897898745	0.922854951	W176596	0.870042315	0.909723604
W170354	0.951300073	0.952191817	W176599	0.816292929	0.858154037
W170381	0.893168842	0.902970795	W176605	0.929736978	0.930386117
W170419	0.997338359	1.000000000	W176616	0.895168075	0.895402993
W170434	1.000000000	1.000000000	W176631	0.815142201	0.843651473
W170443	0.871741771	0.871951834	W176657	0.828384049	0.86268798
W170445	0.756265865	0.802714088	W176678	0.789285226	0.886710716
W170456	0.946111033	0.964927599	W176687	0.927939373	1.000000000
W170465	0.885355662	1.000000000	W176696	1.000000000	1.000000000
W170476	0.852219794	0.873634549	W178008	0.805120632	0.819309245
W170477	0.851654724	0.8730129	W178031	0.829316733	0.887812383
W170479	0.841403752	0.986985828	W178084	0.878653406	0.882405173
W170533	0.895303944	0.913133755	W178101	1.000000000	1.000000000
W170535	0.874693307	0.875628153	W178123	1.000000000	1.000000000
W170540	0.872532989	0.930648082	W178139	0.812685619	0.826500874
W170541	0.903948117	0.916675401	W178202	0.871403579	0.879208391
W170588	0.819232261	0.857585728	W178213	0.969715775	0.994162007
W170618	0.953856764	0.957907806	W178279	0.885028664	0.897974572
W170629	0.874124927	0.891811964	W178293	0.897083796	0.898617833
W170663	1.000000000	1.000000000	W178297	0.921936937	0.96996769
W170774	1.000000000	1.000000000	W178354	0.900415541	0.92135467
W170975	0.990059599	0.99138385	W178380	0.971070302	1.000000000
W174196	0.876135375	0.905977712	W178488	0.786077578	0.821540205
W174226	0.913404081	0.923084858	W178497	0.966649595	1.000000000
W174266	0.887554906	0.916342913	W178541	1.000000000	1.000000000
W174271	0.990955974	1.000000000	W178597	0.909016601	0.93843765
W176116	0.883103994	0.894151542	W178656	0.794962804	0.864654328
W176137	0.982759541	0.98900889	W178672	0.792822275	0.848194581
W176145	1.000000000	1.000000000	W178703	0.918540265	0.938985617
W176158	0.881209117	0.907829703	W178741	0.854989062	0.877909319
W176159	0.874899471	0.88091856	W178744	0.911219953	0.940515158
W176337	0.955696293	1.000000000	W178791	0.942099031	0.95877166

TABLE B.9: Efficiency scores of stores in the North West region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W170108	0.866676769	0.968186632	W174257	0.964493263	0.96467344
W170124	0.860166551	0.897718123	W176118	0.905813748	0.925816743
W170126	0.941206391	0.963498058	W176124	0.941037949	1.000000000
W170134	0.843471836	0.884067157	W176134	0.882421817	0.887303746
W170136	0.881574823	0.882520136	W176171	0.930479391	0.940768208
W170142	0.705141044	0.750594612	W176192	0.967516085	0.980990842
W170155	0.99896716	1.000000000	W176225	0.932319504	0.93999002
W170169	0.8964077	0.930333596	W176299	0.958583441	0.958734624
W170174	0.836705155	0.839340844	W176303	0.987241101	0.989250271
W170195	0.933837323	0.947730523	W176342	0.949875702	0.976137731
W170227	0.896517798	0.90991516	W176389	0.907387622	0.960171367
W170235	0.968373647	0.992388894	W176416	0.896446899	0.901190424
W170241	0.852515264	0.919193419	W176441	0.839915062	0.940163537
W170254	0.880785224	0.881003754	W176473	0.940402395	1.000000000
W170280	0.84103686	0.98156519	W176511	0.771071541	0.835112718
W170313	0.843702586	0.8521432	W176523	0.894723208	0.946541015
W170314	1.000000000	1.000000000	W176547	0.950976521	1.000000000
W170316	0.844577584	0.88176485	W176594	0.953512471	1.000000000
W170322	0.850504281	0.909179204	W176635	0.92132622	0.926172337
W170324	1.000000000	1.000000000	W176660	0.866042038	0.94491659
W170325	0.974211741	0.976095177	W176664	1.000000000	1.000000000
W170347	0.884235627	1.000000000	W176681	0.907862394	0.934917038
W170357	0.91889324	0.971249142	W178045	0.924598153	0.989767187
W170367	0.755825508	0.756052786	W178056	0.894067384	1.000000000
W170387	0.90330745	0.936156519	W178081	0.915098261	0.931584429
W170395	0.914366183	0.917376673	W178109	0.926054314	0.926111768
W170397	0.842650335	0.862641071	W178113	0.944252628	0.997468467
W170421	1.000000000	1.000000000	W178224	0.911955685	1.000000000
W170422	0.746248066	0.749480855	W178332	0.891080619	0.89168481
W170424	0.904634406	0.952943222	W178333	0.931271267	1.000000000
W170444	0.811680676	0.81669893	W178367	0.835661502	0.896013807
W170469	0.744770113	0.772629977	W178385	0.912503223	0.912650894
W170487	0.878429535	0.886774222	W178404	0.898842642	0.9111345
W170489	0.895237497	0.89564199	W178424	1.000000000	1.000000000
W170509	0.89431401	0.92381631	W178454	0.899514716	0.921839895
W170512	0.904758104	1.000000000	W178461	0.980252024	0.986152437
W170525	0.880945881	0.883758078	W178603	0.939249013	0.940534054
W170561	1.000000000	1.000000000	W178611	0.961403848	1.000000000
W170572	0.890249968	0.89037179	W178648	0.999372445	1.000000000
W170614	1.000000000	1.000000000	W178654	0.948074706	0.968145616
W170628	0.939286061	0.955214031	W178690	1.000000000	1.000000000
W170648	1.000000000	1.000000000	W178753	0.979648618	0.980147492
W170690	0.878866676	0.87913477	W178760	0.977956619	1.000000000
W170909	0.834389024	0.843066142	W178768	0.87973924	0.929301666
W170922	1.000000000	1.000000000	W178781	1.000000000	1.000000000
W170965	1.000000000	1.000000000			

TABLE B.10: Efficiency scores of stores in the Free State region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W170181	0.934377872	0.960527246	W176462	0.837271978	0.891201292
W170208	0.888740137	0.907981074	W176467	0.906090899	0.930842797
W170210	0.966715819	1.000000000	W176468	0.905958122	0.933794064
W170217	0.830093438	0.914456118	W176494	0.863788556	0.8648985
W170220	0.881681271	0.919583819	W176520	1.000000000	1.000000000
W170222	0.913695076	0.914211945	W176531	0.859998622	0.86875902
W170233	0.926468623	1.000000000	W176533	0.921862584	0.923935411
W170259	0.919167871	0.920040806	W176545	0.878670439	0.894663097
W170285	0.952895324	1.000000000	W176555	0.925221355	0.93606227
W170296	0.909387577	0.916273273	W176564	0.873797267	0.917736633
W170304	0.957218282	0.978448323	W176580	0.928300825	0.951892909
W170311	0.837462087	0.880609442	W176601	0.978691247	0.982780975
W170321	0.916848125	0.92739459	W176607	0.866200001	0.915157659
W170349	1.000000000	1.000000000	W176608	0.845519817	0.849668584
W170414	0.905726486	1.000000000	W176612	0.975070026	1.000000000
W170437	1.000000000	1.000000000	W176630	0.933504327	0.935974209
W170464	0.928453862	0.999146482	W176667	0.782396319	0.86006734
W170474	0.857160379	0.861011765	W176668	1.000000000	1.000000000
W170482	0.953998162	0.967723001	W176684	0.976538298	1.000000000
W170511	0.928252243	0.929414401	W176686	0.950521412	0.97906185
W170546	0.843207441	0.851418947	W176700	0.80287916	1.000000000
W170583	0.948216224	1.000000000	W176701	0.935145288	0.961526064
W170605	0.825383311	0.85137944	W176710	1.000000000	1.000000000
W170643	0.802280113	0.847297338	W178026	0.852831784	0.911576628
W170662	0.971199319	0.989175428	W178063	1.000000000	1.000000000
W170688	0.879206353	0.938498268	W178073	0.890525487	0.892664935
W170772	0.806871167	0.870550616	W178099	1.000000000	1.000000000
W170971	0.846782428	0.858149177	W178107	0.911262819	0.940846111
W170976	0.775452641	0.8268978	W178225	1.000000000	1.000000000
W174170	1.000000000	1.000000000	W178281	0.909255318	0.978660094
W174256	0.871185648	0.971007151	W178359	0.815011335	0.838712147
W174259	0.986376541	0.993965169	W178375	0.950733473	0.954603898
W174527	0.761675975	0.853808733	W178401	0.906672212	0.96189809
W176165	0.899821392	0.957660356	W178495	0.942541242	0.947363733
W176211	0.958499902	0.959278738	W178509	0.79253745	0.800272672
W176236	1.000000000	1.000000000	W178534	0.960347829	0.965002507
W176240	1.000000000	1.000000000	W178573	0.871890671	0.931348172
W176302	0.805201761	0.860460604	W178649	0.9433598	1.000000000
W176334	0.97558917	0.997269661	W178694	0.893105967	0.894655699
W176357	0.881430014	0.918649065	W178712	0.855012026	0.897967786
W176369	1.000000000	1.000000000	W178722	0.955206771	1.000000000
W176409	0.861424956	0.867766364	W178743	1.000000000	1.000000000
W176415	0.832144439	0.891813171	W178773	0.981379097	1.000000000
W176431	0.912271663	0.912653427	W178782	0.800414307	0.848255048
W176449	0.855669118	0.856317765	W178794	0.886415211	0.965917198
W176450	0.923467605	0.928503213			

TABLE B.11: Efficiency scores of stores in the Lesedi region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
0137	1.000000000	1.000000000	W166464	0.986744181	0.997774175
0224	0.922875078	0.94201213	W166472	0.898153959	0.948019915
0253	0.888356072	0.935365182	W166481	0.941372363	0.954242459
0352	0.849810138	0.915710271	W166493	0.991467702	0.999339595
0353	1.000000000	1.000000000	W166500	1.000000000	1.000000000
0356	0.967597844	1.000000000	W166504	0.907123922	0.948931905
0452	0.921286193	0.944751342	W166508	0.946461489	0.998304329
0473	0.865632891	0.937478563	W166513	0.932147057	0.969919184
0495	1.000000000	1.000000000	W166521	0.87449187	0.915471517
0501	0.842421455	0.899228511	W166535	0.864302552	0.945614366
0548	0.831604765	0.854624667	W166537	0.952598641	0.953221893
0616	0.907330402	0.952920449	W166579	0.91111395	0.94749572
0651	0.925707945	0.982600089	W166581	0.88058121	0.934071211
0787	0.803197554	0.88128165	W166586	0.85171014	0.902802412
0943	0.956107876	0.95865878	W166593	1.000000000	1.000000000
0949	0.975613675	0.982043152	W166663	1.000000000	1.000000000
0960	0.869333486	0.923065237	W166676	1.000000000	1.000000000
0987	1.000000000	1.000000000	W166689	0.886316438	1.000000000
0996	0.987435136	0.998219589	W166690	1.000000000	1.000000000
W166117	0.993089326	1.000000000	W166702	0.822015705	0.914550251
W166120	0.795258066	0.873210983	W168086	0.925280191	0.953683931
W166132	0.832303677	0.932489683	W168211	0.919960077	0.964742041
W166133	0.936620314	0.961496526	W168215	1.000000000	1.000000000
W166140	0.822964669	0.851103903	W168217	0.989216754	0.995311931
W166167	0.798793003	0.902462606	W168235	0.947627203	0.955365169
W166177	0.898917293	0.924708461	W168266	0.898211882	0.945533058
W166182	0.905680724	0.913942073	W168280	0.853443584	0.897652211
W166200	0.955630224	0.973066188	W168376	1.000000000	1.000000000
W166206	0.920866902	0.976223871	W168408	0.921875964	0.925845778
W166213	0.999669441	1.000000000	W168432	1.000000000	1.000000000
W166214	0.831759772	0.84443138	W168460	0.943545376	0.946651209
W166218	0.94626097	0.978280957	W168486	0.97572243	0.988597847
W166219	0.967959296	0.971345439	W168490	0.943495148	0.951234979
W166272	1.000000000	1.000000000	W168502	0.895303362	0.96023373
W166281	0.988591982	1.000000000	W168514	0.937667668	1.000000000
W166320	0.891808844	0.920020474	W168559	0.959952044	0.993757853
W166340	1.000000000	1.000000000	W168574	0.936692273	0.964913431
W166346	0.842988721	0.921718478	W168586	0.882220819	0.93781949
W166352	0.970729061	0.977557423	W168652	0.953459487	0.976800352
W166353	0.879258111	0.966913876	W168660	1.000000000	1.000000000
W166356	0.941787647	0.964669469	W168677	0.889291574	0.934223807
W166395	0.975849354	0.975976525	W168695	0.960843082	1.000000000
W166400	0.97617245	1.000000000	W168724	0.884743167	0.9491567
W166401	1.000000000	1.000000000	W168727	1.000000000	1.000000000
W166405	0.890548685	0.953181604	W168728	0.934486189	1.000000000
W166424	0.92971352	0.960847607	W168758	1.000000000	1.000000000
W166442	0.876778822	0.937523914	W168775	0.797501555	0.82224966
W166451	0.963850268	0.976802245	W168778	0.880958702	0.977116001
W166452	0.976861574	0.989275386	W168793	0.927005576	0.957324078

TABLE B.12: Efficiency scores of stores in the Gauteng region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160132	0.733734886	0.88794288	W166439	0.879583593	0.90862643
W160164	0.994637913	0.996907741	W166445	0.875696011	0.889605112
W160201	0.943278748	0.943496889	W166459	0.982730488	1.000000000
W160211	0.983968328	0.992621976	W166484	0.949073234	0.95154172
W160218	0.974195952	0.981320591	W166488	0.922856172	0.927945291
W160249	0.894749991	0.938375262	W166510	0.941334743	0.94815725
W160272	0.886944813	0.933940733	W166515	0.814209285	0.931959138
W160310	0.799027065	0.909604345	W166518	1.000000000	1.000000000
W160318	0.925630554	0.94743217	W166527	0.75978952	0.855662305
W160338	1.000000000	1.000000000	W166538	0.905966878	0.910637752
W160345	0.962252896	0.964157137	W166544	1.000000000	1.000000000
W160401	0.938383654	0.951126472	W166568	1.000000000	1.000000000
W160425	0.7915329	0.854610048	W166573	0.789483827	0.918594117
W160440	0.781909625	0.859639942	W166617	0.792072989	0.878081075
W160503	0.945165802	0.97351228	W166627	1.000000000	1.000000000
W160582	0.8199259	0.850721949	W166633	0.965214483	1.000000000
W160603	0.87102803	0.95579866	W166639	0.891183618	0.934342053
W160607	1.000000000	1.000000000	W166651	0.885002638	0.928338191
W160620	0.863165713	0.926697923	W166658	0.963966438	0.988279088
W160642	0.681857585	0.823231812	W166698	1.000000000	1.000000000
W160681	0.838109938	0.943151671	W166704	1.000000000	1.000000000
W160696	0.8006978	0.878673849	W168032	0.981474841	0.999071361
W160914	0.861242835	0.916172706	W168035	0.785354954	0.881513852
W160918	0.969178199	0.970373615	W168095	0.805291208	0.971969764
W160919	0.985046839	0.986325488	W168102	1.000000000	1.000000000
W160931	0.884579064	0.928045508	W168103	0.820732475	0.941587869
W160935	0.876952422	0.883418906	W168104	0.919746919	0.941142666
W160967	0.946052507	0.949280861	W168105	1.000000000	1.000000000
W160993	0.831429868	0.868299126	W168212	0.653450835	0.83000457
W164230	0.776596807	0.811377725	W168251	0.939826271	0.940962005
W164258	0.838061694	0.846373768	W168317	0.987341644	0.990186152
W164509	0.792825049	0.87281323	W168326	1.000000000	1.000000000
W164525	0.94038116	0.95741642	W168338	0.860159803	0.929701264
W166101	0.844341291	0.954699911	W168387	0.991787094	0.999254675
W166178	0.859811194	0.895354418	W168417	0.980826026	0.987162485
W166231	0.990422931	1.000000000	W168427	0.700949059	0.848012685
W166232	0.820999796	0.925612943	W168527	0.876320911	0.886543829
W166247	1.000000000	1.000000000	W168535	0.92836296	0.947738087
W166265	0.910202289	0.974218023	W168536	0.728456077	0.871067244
W166266	0.95438679	0.979895561	W168548	0.930749498	0.948403261
W166332	0.92969465	0.93929526	W168549	0.982654455	1.000000000
W166336	1.000000000	1.000000000	W168590	0.95300683	0.964324128
W166368	0.740425681	0.884338333	W168663	0.954963554	0.965539882
W166378	0.800943488	0.84257127	W168664	0.96961888	1.000000000
W166382	0.869688101	0.883564596	W168670	0.883567076	0.891147863
W166385	0.959825095	0.986573061	W168715	0.972653076	0.981228337
W166403	0.882886939	0.89395818	W168737	0.970581231	1.000000000
W166430	0.916072331	0.9539512	W168779	0.891240012	0.900530382
W166437	0.938357534	0.942149784	W168780	0.87764993	0.894659214

TABLE B.13: Efficiency scores of stores in the Limpopo region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160141	0.862740837	0.869617422	W166548	0.95512486	1.000000000
W160149	0.835681622	0.907621753	W166549	0.817467619	0.877178518
W160185	0.909167599	0.933306742	W166576	0.840044264	0.923194696
W160276	0.927715999	0.983222474	W166577	0.802369484	0.865859705
W160359	0.901480944	0.920540361	W166598	0.928331213	0.980391468
W160442	0.803263439	0.861138266	W166615	1.000000000	1.000000000
W160458	0.936419828	1.000000000	W166622	1.000000000	1.000000000
W160461	1.000000000	1.000000000	W166632	0.840574388	0.894413021
W160490	0.956905155	0.967219882	W166647	0.991462031	1.000000000
W160532	1.000000000	1.000000000	W166653	0.971397848	0.972518901
W160566	0.934029598	0.934105048	W166655	0.943554128	0.948344734
W160580	1.000000000	1.000000000	W166682	1.000000000	1.000000000
W160604	0.888494461	0.916889649	W168006	0.923810201	0.972298558
W160693	0.88507078	0.943604521	W168011	0.796626666	0.817810275
W160695	1.000000000	1.000000000	W168013	0.840959266	0.881043537
W160735	0.923883621	0.942634888	W168055	0.87122446	0.936479418
W160736	0.858203541	0.908815523	W168238	0.807307784	0.909143832
W160737	1.000000000	1.000000000	W168239	0.880650635	0.959701135
W160740	1.000000000	1.000000000	W168247	0.959778907	0.97574892
W160742	1.000000000	1.000000000	W168261	0.794504896	0.878336797
W160743	0.831986307	0.834384365	W168262	0.917452157	1.000000000
W160746	0.879223038	0.922112362	W168273	0.879085528	0.907425494
W160748	0.799780201	0.879768762	W168285	0.942479959	0.992311304
W160779	1.000000000	1.000000000	W168289	0.941751217	0.948298094
W160785	1.000000000	1.000000000	W168291	0.845217428	0.861580676
W160925	0.92074586	0.954066006	W168303	0.868908946	0.869218824
W160941	0.935719345	0.939193269	W168321	0.925351965	0.926926142
W164278	0.903022208	0.91169783	W168337	0.874664813	0.907200594
W164280	0.805101896	0.876556466	W168352	0.8924718	0.931956803
W166130	0.808968917	0.862087185	W168353	0.945030092	0.967647019
W166157	0.871396902	0.87413679	W168356	0.89371731	0.963762571
W166164	0.792848761	0.852609754	W168425	0.868906234	0.908851036
W166181	0.883517962	0.888651278	W168426	0.869632455	0.873632611
W166189	1.000000000	1.000000000	W168449	0.847073548	0.883455714
W166202	0.901036907	0.920851399	W168462	0.8639252	0.891795877
W166203	0.839757876	0.923417848	W168463	0.983040001	1.000000000
W166217	0.969027194	0.973750162	W168465	0.884539815	0.955899999
W166226	1.000000000	1.000000000	W168477	1.000000000	1.000000000
W166230	0.846373082	0.855158354	W168522	1.000000000	1.000000000
W166244	0.916317235	0.96557081	W168525	0.939492401	0.949240787
W166301	0.848545474	0.911401764	W168545	0.85826072	0.919893919
W166328	0.928900324	0.987644728	W168587	0.997573321	1.000000000
W166344	0.906345026	0.952278636	W168666	0.850826222	0.875061288
W166379	0.857238581	0.881185105	W168682	1.000000000	1.000000000
W166383	0.887130998	0.918512925	W168685	0.822870283	0.905068697
W166387	0.977264033	0.984630064	W168688	1.000000000	1.000000000
W166407	0.932013809	1.000000000	W168692	1.000000000	1.000000000
W166434	0.811930442	0.832379706	W168736	0.89188207	0.985706732
W166457	0.955369898	0.956092745	W168746	0.916324626	0.917956395
W166476	1.000000000	1.000000000	W168766	0.807438772	0.915158192
W166501	0.907237339	0.921549814	W168776	1.000000000	1.000000000
W166529	0.889508892	0.891056333	W168784	0.977337487	0.999884055
W166539	0.898037321	0.898175903			

TABLE B.14: Efficiency scores of stores in the Thekwini region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160178	0.965875171	0.975082733	W166522	0.921366715	0.921423825
W160229	1.000000000	1.000000000	W166530	0.881063225	1.000000000
W160274	1.000000000	1.000000000	W166536	0.896793105	0.903275796
W160312	0.990975007	0.99238309	W166552	1.000000000	1.000000000
W160364	1.000000000	1.000000000	W166572	0.956794731	0.96746793
W160389	0.878158073	0.886224924	W166585	0.920087064	0.945845603
W160398	0.895808967	0.924427426	W166602	0.859931498	0.893664096
W160408	0.917843753	0.958416653	W166609	0.894663173	0.895926031
W160436	0.942003364	0.950077156	W166613	1.000000000	1.000000000
W160460	0.920059785	0.923365576	W166614	0.976984265	0.979628193
W160483	0.97113394	0.971574152	W166629	1.000000000	1.000000000
W160498	0.908764648	0.913250339	W166634	0.971218193	0.971219855
W160543	0.925981802	0.934716491	W166640	1.000000000	1.000000000
W160619	0.906341494	0.934963127	W166652	1.000000000	1.000000000
W160640	0.888998943	0.913403554	W166659	0.798134817	0.836722435
W160778	0.914822432	0.932713252	W166665	1.000000000	1.000000000
W160792	1.000000000	1.000000000	W166672	0.857590646	0.937925273
W160911	1.000000000	1.000000000	W166673	1.000000000	1.000000000
W160913	0.926402691	0.93230167	W166680	0.992424318	1.000000000
W160936	0.906450863	0.934075294	W166691	1.000000000	1.000000000
W160953	0.99702385	1.000000000	W166692	0.991036226	1.000000000
W164132	0.861500062	0.885465952	W166693	0.887115318	0.945004417
W164208	0.94436574	0.953975482	W166705	0.839515447	0.841882279
W166102	0.960546682	0.960805288	W168024	1.000000000	1.000000000
W166123	0.92526075	0.942681147	W168082	0.886902458	0.89518259
W166139	0.968958042	0.970294767	W168094	0.863003176	0.876520018
W166173	0.992921335	0.995855658	W168096	0.949914518	0.952002237
W166175	0.965031763	0.977957693	W168097	1.000000000	1.000000000
W166176	0.912347011	1.000000000	W168098	0.906083015	0.935567612
W166209	0.890279882	0.920943689	W168128	0.91089118	0.918898822
W166216	0.934431998	0.937793685	W168223	0.925963159	0.926268226
W166224	0.95074032	0.957227433	W168268	0.797747473	0.832829234
W166280	0.999128991	1.000000000	W168287	0.903283812	0.916138218
W166314	0.902017493	0.942723492	W168288	0.904177327	0.91480473
W166323	0.956854386	0.969623558	W168301	0.83286148	0.856243241
W166325	0.959557134	0.968822562	W168319	0.820191627	0.845325374
W166326	0.837146984	0.867039575	W168336	0.950124673	0.994111685
W166327	0.834149635	0.872715221	W168402	0.893182602	0.895600007
W166339	0.978294827	0.987691812	W168409	0.90976242	0.912734911
W166345	0.859600272	0.915812428	W168411	0.809953366	0.813910675
W166381	1.000000000	1.000000000	W168434	0.910043312	0.957613851
W166411	0.937895134	0.940563131	W168494	0.937185958	0.947228434
W166420	0.942425938	0.942435856	W168513	0.933764166	0.934790958
W166423	0.961301199	0.97897842	W168558	0.963605982	0.969686636
W166433	0.908366789	0.931432757	W168588	0.939310449	0.941213931
W166448	1.000000000	1.000000000	W168646	0.901884392	0.914411222
W166463	0.966220256	0.994694463	W168655	0.977852476	1.000000000
W166471	0.924972465	0.927244304	W168697	1.000000000	1.000000000
W166498	0.988011056	0.996761979	W168717	0.969758183	0.971244564
W166499	0.972901331	0.992641858	W168738	0.958420538	0.974864933
W166503	0.868454791	0.873082114	W168783	0.930781085	0.945339989
W166507	0.987571912	1.000000000	W168790	0.924328459	0.972486494
W166514	0.914041358	0.939132216	W168795	0.915788434	0.932649885
W166517	0.984115351	0.992172028			

TABLE B.15: Efficiency scores of stores in the Tugela region of season W17 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160105	0.643071566	0.841155574	W166444	0.671730623	0.67314492
W160139	0.344435596	0.421975948	W166485	0.393313299	0.417774778
W160197	0.565752725	0.694466022	W166487	1.000000000	1.000000000
W160198	0.281364904	0.284214974	W166516	0.766237041	0.779805792
W160202	0.133606665	0.202313615	W166541	0.147747376	0.147915059
W160231	0.564953906	0.970494488	W166554	0.549503862	0.644658863
W160328	0.215385426	0.228638187	W166584	0.601573191	0.73229125
W160448	0.789605119	0.803232062	W166638	1.000000000	1.000000000
W160478	0.46544398	0.670725726	W166649	0.327949089	0.510927642
W160734	0.888759903	0.88899034	W166666	1.000000000	1.000000000
W160982	1.000000000	1.000000000	W166674	0.506793949	0.568058498
W164168	0.092547776	0.093097026	W166679	1.000000000	1.000000000
W164172	0.364075376	0.504654738	W168263	0.107137559	0.108781103
W164203	0.823786799	0.839072446	W168510	0.831779067	0.929544943
W1605617	0.006691396	1.000000000	W168515	0.837273533	0.837633332
W1605672	0.006691396	1.000000000	W168531	0.524643542	0.606404724
W166201	0.129118686	0.1494638	W168683	0.337468061	0.337468061
W166286	0.616989843	0.70275732	W168732	0.247432296	0.461879083
W166324	1.000000000	1.000000000	W168752	0.328334251	0.487379584

TABLE B.16: *Efficiency scores of stores in the Southern Namibia region of season W16 under CRS and VRS.*

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160167	0.336465255	1.000000000	W166624	0.825365574	0.85337182
W160219	0.551459679	0.564154346	W166625	1.000000000	1.000000000
W160256	0.764433712	0.787039154	W166626	0.272251182	0.279260441
W160297	0.362121649	0.457547802	W166637	0.701913264	0.744772829
W160323	0.582171084	0.603069229	W166656	0.223314047	0.261097216
W160326	0.494439544	0.494710602	W166669	1.000000000	1.000000000
W160432	0.485294131	0.521780006	W166688	0.993161356	1.000000000
W160447	0.464243345	0.623981764	W168271	0.255175267	0.261497063
W160467	0.707896924	0.73766707	W168272	1.000000000	1.000000000
W160507	1.000000000	1.000000000	W168298	0.231689577	0.235889908
W160981	1.000000000	1.000000000	W168340	0.704639446	0.757211652
W160984	0.29240085	0.306087324	W168370	0.523547177	0.554758064
W166144	1.000000000	1.000000000	W168382	0.699819758	0.745287807
W166174	0.512688782	0.532400962	W168391	0.532791683	1.000000000
W166229	0.319859903	0.339324087	W168393	0.266395035	0.285300426
W166469	0.815909432	1.000000000	W168412	0.910110555	1.000000000
W166506	0.686603122	0.733200091	W168420	0.882990333	1.000000000
W166519	0.639484155	0.64805613	W168493	1.000000000	1.000000000
W166543	0.101669289	0.105368723	W168499	1.000000000	1.000000000
W166561	0.537526759	0.54390645	W168520	0.627423802	0.647039676
W166571	0.446459192	0.618292586	W168521	0.6251834	0.757895069
W166582	1.000000000	1.000000000	W168529	0.229996551	0.237536821
W166600	1.000000000	1.000000000	W168539	0.884027678	1.000000000
W166620	0.770256348	0.770512985	W168546	0.493992902	1.000000000
W166623	0.451880782	0.465974962			

TABLE B.17: *Efficiency scores of stores in the Northern Namibia region of season W16 under CRS and VRS.*

Store ID	θ_{CRS}	θ_{VRS}
W160115	1.000000000	1.000000000
W160199	1.000000000	1.000000000
W160428	0.954264969	1.000000000
W160520	0.901497624	1.000000000
W160536	1.000000000	1.000000000
W160597	0.693770115	1.000000000
W160634	0.671901186	0.674609598
W160907	0.476941091	0.477573391
W160964	1.000000000	1.000000000
W160978	0.65107025	0.677212892

Store ID	θ_{CRS}	θ_{VRS}
W164530	1.000000000	1.000000000
W166193	1.000000000	1.000000000
W166532	1.000000000	1.000000000
W168089	0.833929643	0.858977349
W168292	1.000000000	1.000000000
W168323	0.810940635	0.859448629
W168346	1.000000000	1.000000000
W168357	1.000000000	1.000000000
W168512	1.000000000	1.000000000

TABLE B.18: Efficiency scores of stores in the Swaziland region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}
W160257	0.242908711	0.29708482
W160309	0.549606997	0.552073498
W160402	0.652631722	0.676707045
W160549	0.320863414	0.327521146
W160551	1.000000000	1.000000000
W160552	0.473125076	0.478754565
W160554	0.717904569	0.722054494
W160555	0.817314426	1.000000000
W160556	1.000000000	1.000000000
W160575	0.924570286	0.942555265
W160905	1.000000000	1.000000000
W160926	0.649197083	0.90300423
W160945	0.517406597	0.788172892
W166105	0.882623967	1.000000000
W166119	0.583377144	0.641171592
W166135	0.80642706	0.964094111
W166208	1.000000000	1.000000000
W166234	0.853869245	0.925711107
W166264	0.612298743	0.615275922
W166295	0.661472001	0.747803906
W166341	0.727328582	1.000000000
W166361	0.810669444	0.813173177
W166435	0.827945862	1.000000000
W166440	0.472164766	0.54657932
W166447	0.671079916	0.686208033
W166456	1.000000000	1.000000000
W166470	0.616706112	0.799014638
W166525	0.46502814	0.549305307
W166551	1.000000000	1.000000000
W166560	0.646552349	0.652304305
W168087	0.614496629	1.000000000
W168204	0.632419982	0.70033958
W168226	1.000000000	1.000000000
W168227	1.000000000	1.000000000

Store ID	θ_{CRS}	θ_{VRS}
W168274	0.669644939	0.699475677
W168284	0.981507858	0.988016454
W168310	0.524424276	0.576377157
W168311	0.65130177	0.664517344
W168325	0.641082007	0.667383181
W168348	0.549902899	0.610493013
W168349	0.828405344	0.861503103
W168350	0.876149041	1.000000000
W168373	0.206276263	0.206601179
W168374	0.141451017	0.15608392
W168396	1.000000000	1.000000000
W168397	0.240903953	0.273214989
W168398	0.536565519	0.71580503
W168399	0.94188524	0.94188524
W168414	0.421649579	0.428649274
W168415	0.510887344	0.575700413
W168476	0.623071256	0.648808051
W168501	0.783805355	1.000000000
W168537	1.000000000	1.000000000
W168538	0.389297554	0.397641947
W168569	0.743002211	0.839378026
W168571	0.284409562	0.348447698
W168593	0.523449368	0.583034855
W168600	0.770301282	0.819314793
W168665	0.584847764	0.633478908
W168706	0.242954933	0.292017382
W168730	0.312388554	1.000000000
W168731	0.546320768	0.548728276
W168734	0.90535554	1.000000000
W168747	0.657556779	1.000000000
W168749	0.983603439	1.000000000
W168750	0.386436797	0.427579039
W168786	0.618330794	0.642416247
W168797	1.000000000	1.000000000

TABLE B.19: Efficiency scores of stores in the Botswana region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160104	0.543376209	0.552157923	W160730	0.313681949	0.36909163
W160109	0.356337381	0.524648039	W160732	0.471138478	0.473021176
W160111	1.000000000	1.000000000	W160780	0.613171049	0.619081244
W160144	0.232670427	0.31746463	W160789	0.420945541	0.427576882
W160148	0.697107972	0.72457198	W160970	0.696996623	0.704587078
W160151	0.696569629	0.701125981	W164101	0.6365712	0.64310975
W160154	0.939948711	0.959277319	W164244	1.000000000	1.000000000
W160156	0.573283304	0.692850374	W166136	0.703522687	0.726776535
W160173	0.731726624	0.733312951	W166146	0.558429093	0.608160465
W160184	1.000000000	1.000000000	W166161	0.641371925	0.651890458
W160188	0.605135183	0.613202019	W166170	0.632985629	0.858824351
W160190	1.000000000	1.000000000	W166268	0.483542751	0.493646852
W160206	0.980383577	1.000000000	W166277	0.810258573	0.860133743
W160226	0.526492861	0.53426627	W166279	0.560040638	0.574526272
W160228	0.291949027	0.297392196	W166317	0.512491671	0.517938862
W160234	0.521330905	0.526368124	W166349	0.528001057	0.53035896
W160247	0.390652917	0.397367514	W166351	0.590320872	0.598385213
W160248	0.641601419	0.64432963	W166355	0.383151311	0.389065723
W160251	0.421077704	0.424514418	W166414	0.612368264	0.637861129
W160270	0.73671328	0.742323923	W166455	0.664880782	0.676996448
W160290	1.000000000	1.000000000	W166465	0.692077534	0.812297295
W160333	0.159561241	0.169789308	W166534	0.411165217	0.415299022
W160341	0.278109965	0.281537998	W166575	0.500102197	0.5063119
W160344	0.350295144	0.355666429	W166590	0.306349165	0.308848287
W160361	0.614386686	0.62166073	W166643	0.500427164	1.000000000
W160366	0.260015365	0.262925807	W166670	1.000000000	1.000000000
W160369	0.35525809	0.375530412	W166671	1.000000000	1.000000000
W160372	0.763219199	0.767929029	W168018	0.384710628	0.386878838
W160382	0.90915793	0.925464398	W168042	0.179818245	0.180369399
W160386	0.753131419	0.819640883	W168046	0.564231152	0.567296547
W160391	0.459813925	0.645335276	W168092	0.274657062	0.285628264
W160405	0.578360075	0.584790421	W168133	0.41800649	0.420150704
W160411	0.59187737	0.717706412	W168365	0.56415136	0.564722622
W160417	0.437958144	0.440922898	W168388	0.385744799	0.388388507
W160457	0.46365143	0.468273097	W168419	0.241595485	1.000000000
W160494	0.498635329	0.517262542	W168487	0.325164532	0.329260515
W160505	0.674978386	0.681031795	W168674	0.629921651	0.632394449
W160565	0.569452466	0.715954455	W168679	0.223135042	0.274552854
W160570	0.542953628	0.549210462	W168696	0.70855844	0.71708727
W160621	0.74758481	0.75406827	W168726	0.452438899	0.617040254
W160723	0.44953332	0.451848293	W168735	0.511311609	0.513655432
W160724	0.577419157	0.615835573	W168792	1.000000000	1.000000000
W160727	0.5812188	0.585603178			

TABLE B.20: Efficiency scores of stores in the Cederberg region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160221	0.726288975	0.726288975	W166422	0.50569156	0.50569156
W160255	1.000000000	1.000000000	W166432	0.856129179	0.856654613
W160271	0.59215967	0.59440185	W166460	0.455539668	0.45670469
W160291	0.615557143	0.642987414	W166461	0.437842267	0.438112126
W160303	0.458871092	0.466144642	W166480	0.415710046	0.416003576
W160335	0.423897477	0.42509283	W166490	0.430629346	0.470736347
W160430	0.585548679	0.587102262	W166495	0.709218709	0.765733018
W160542	0.346456357	0.347247768	W166512	0.84940792	0.897998789
W160630	0.431837579	0.439358293	W166562	0.745784478	0.745784478
W160638	0.524757363	0.540595867	W166565	0.469781159	0.479726107
W160639	1.000000000	1.000000000	W166570	0.319882998	0.319970272
W160674	0.431112431	0.46686697	W166578	0.628935385	0.632616386
W160797	1.000000000	1.000000000	W166588	0.244566932	0.245322603
W160924	0.520061581	0.520957054	W166641	0.484493334	0.48477846
W160989	0.741470912	0.753702703	W166642	1.000000000	1.000000000
W164506	0.57467289	0.595086652	W166654	0.862571025	0.882176179
W166115	0.334321983	0.334370798	W166677	0.811631699	0.876122823
W166127	0.33560034	0.336349226	W168079	0.37579336	0.396603394
W166138	0.446431031	0.4493729	W168114	0.721476066	0.756150356
W166180	0.286817666	0.349449291	W168218	0.464390198	0.471956546
W166184	0.888695564	0.890137181	W168220	0.497130162	0.503557132
W166194	0.326186643	0.327619156	W168258	0.77600715	0.957888073
W166205	0.52939749	0.529640259	W168269	0.328754162	0.352072666
W166220	0.710900154	0.790100531	W168278	0.341288325	0.350444574
W166221	0.438362507	0.456086009	W168306	0.45610649	0.463831254
W166243	0.477275783	0.479597429	W168378	0.407703537	0.414956973
W166246	0.526702587	0.527473057	W168403	0.526891276	0.528070915
W166273	0.364187138	0.372454297	W168430	0.550539585	0.552861236
W166274	0.616081488	0.624927202	W168431	0.582254355	0.599301588
W166321	0.425787371	0.434196586	W168437	0.435827744	0.437862396
W166331	0.230074014	0.230251499	W168438	1.000000000	1.000000000
W166338	0.696330724	0.719680867	W168489	0.514274021	0.514501241
W166358	1.000000000	1.000000000	W168517	0.606649615	0.606977614
W166363	1.000000000	1.000000000	W168592	0.473059333	0.557878399
W166364	1.000000000	1.000000000	W168667	0.870853592	1.000000000
W166367	0.526943415	0.527170851	W168668	0.476646625	0.477601348
W166371	0.40505593	0.405795379	W168671	0.477303001	0.477956683
W166376	0.508549068	0.509517319	W168673	0.433697809	0.434388119
W166377	0.783203114	0.793009215	W168687	0.325275962	0.339977488
W166384	0.968986166	1.000000000	W168689	0.398678348	0.401244625
W166386	0.762161668	0.79152987	W168739	0.535257753	0.536008754
W166390	0.799059975	0.799059975	W168767	0.841754076	0.842038065
W166402	0.440034139	0.440577336	W168772	0.507323977	0.507683917

TABLE B.21: Efficiency scores of stores in the Kwena region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160146	0.512284593	0.519466752	W166366	0.893811526	0.896806943
W160180	0.600464516	0.613521696	W166372	0.930556702	0.931216072
W160223	0.430063928	0.48288406	W166397	0.719776648	0.810794725
W160267	0.599901284	0.642533015	W166413	0.635166389	0.756462381
W160279	0.620235792	0.637032974	W166421	0.37670345	0.384852908
W160289	0.992246062	0.997707216	W166428	0.466071085	0.488514967
W160307	0.819124722	0.822876569	W166436	0.458136983	0.460778706
W160373	0.60140253	0.602681926	W166443	0.52278682	0.609385032
W160400	0.547323563	0.621898614	W166466	0.608485635	0.623937447
W160409	0.489956179	0.702143094	W166482	0.712070538	0.732790533
W160413	0.746843733	0.812720183	W166483	1.000000000	1.000000000
W160418	0.425564331	0.480323472	W166502	0.41453635	0.435875404
W160435	0.565617107	0.579954135	W166524	0.512956742	0.515426298
W160484	1.000000000	1.000000000	W166526	0.586549462	0.641255938
W160499	0.59617179	0.597340919	W166546	0.585510404	0.637519504
W160544	1.000000000	1.000000000	W166550	0.560413448	0.656646768
W160591	0.527723153	0.531049232	W166553	0.758515026	0.801151013
W160610	0.597437039	0.597871461	W166557	0.95562698	0.956320981
W160666	0.466402187	0.488595955	W166558	0.600477377	0.609194704
W160689	0.784544167	1.000000000	W166559	0.326410896	0.365152894
W160796	0.694341045	0.704696202	W166569	1.000000000	1.000000000
W160958	0.582856936	0.97103527	W166606	1.000000000	1.000000000
W160995	0.722489648	0.775090932	W166619	0.438724497	0.592343215
W164209	1.000000000	1.000000000	W166683	1.000000000	1.000000000
W164255	0.593975306	1.000000000	W168023	0.637638744	0.65718726
W164512	0.607225457	0.608254147	W168134	0.535125769	0.538181126
W166104	0.829487677	1.000000000	W168295	0.780161845	0.833160317
W166122	0.712057687	0.904167442	W168384	0.898229524	0.977567644
W166126	1.000000000	1.000000000	W168450	0.521797385	0.957161162
W166141	1.000000000	1.000000000	W168467	0.842462946	0.917280534
W166163	0.803801075	0.833010434	W168511	0.734923328	1.000000000
W166222	0.58318802	0.657724154	W168516	0.619261476	0.620849205
W166235	0.800231446	0.831391626	W168551	0.422909007	0.516011607
W166241	1.000000000	1.000000000	W168552	0.912450548	1.000000000
W166245	0.646882426	0.762029605	W168553	0.750298326	0.757809553
W166252	0.477967762	0.478873679	W168591	1.000000000	1.000000000
W166269	0.595993264	0.738921645	W168681	0.70372326	0.912234845
W166284	0.441698731	0.491400103	W168698	1.000000000	1.000000000
W166285	0.698527048	0.703695599	W168705	0.466677562	0.525819526
W166330	1.000000000	1.000000000	W168711	0.767771072	0.959055599
W166333	0.529024573	0.604109852	W168755	0.687800106	0.836598427
W166335	0.672955023	0.700361075	W168759	0.599711037	0.721267816
W166365	0.572874511	0.620980052	W168769	0.704435854	0.74815913

TABLE B.22: Efficiency scores of stores in the Emfuleni region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160102	0.561877502	0.679350107	W160955	0.591458785	0.594489138
W160112	0.585718602	0.586764082	W164119	0.45775137	0.459702198
W160113	0.346717333	0.362032424	W164231	0.375713017	0.491221637
W160119	1.000000000	1.000000000	W166128	1.000000000	1.000000000
W160131	0.533534541	0.638711179	W166143	0.599610212	0.604537911
W160150	0.47341274	0.474655757	W166160	0.579245535	0.582245117
W160152	0.739526916	0.745119889	W166162	0.330993566	0.369111281
W160160	0.258895784	1.000000000	W166188	0.773716293	1.000000000
W160172	1.000000000	1.000000000	W166215	0.53236492	0.694281491
W160179	0.741862784	0.770982811	W166227	0.912343772	0.91883599
W160182	0.312544106	0.337836071	W166248	0.456821312	0.459186929
W160183	0.468316538	0.519157399	W166287	0.591185032	0.592345217
W160194	1.000000000	1.000000000	W166294	0.478021647	0.487169494
W160215	0.559301935	0.606615391	W166308	0.360103405	0.360956616
W160225	0.798348752	0.838432405	W166311	0.273351058	0.35438904
W160236	0.465709883	0.467042779	W166322	0.528034353	0.529113768
W160245	0.829048197	1.000000000	W166360	0.383301246	0.425551636
W160268	0.788816932	0.790253862	W166375	0.50465537	0.669068009
W160288	0.483754101	0.584591231	W166399	0.466964527	0.470251599
W160320	0.250192118	0.250915216	W166406	0.406700942	0.407643857
W160340	0.346613085	0.446159852	W166426	1.000000000	1.000000000
W160371	0.632087345	0.696886713	W166438	0.37312578	0.373846092
W160374	0.450605538	0.451619088	W166458	0.407575831	0.410837519
W160390	0.527499156	0.697266055	W166478	0.457087521	0.458395738
W160392	0.42421085	0.470618136	W166610	0.65641501	0.788494633
W160396	0.347617525	0.420077384	W166645	0.347369705	0.349151659
W160406	0.472608705	0.504930944	W166661	0.473202448	0.475852987
W160492	0.686272571	0.740403713	W168012	0.516426559	0.519138656
W160513	0.695097291	1.000000000	W168059	0.466682393	0.564808115
W160592	0.477765151	0.501942591	W168065	0.500176717	0.501599789
W160594	0.288555163	0.290049425	W168067	0.347149958	0.348830961
W160598	0.693138127	1.000000000	W168111	1.000000000	1.000000000
W160599	0.389938616	0.421280997	W168315	0.432338192	0.445417472
W160601	0.522816669	0.73212655	W168377	0.484439985	0.486345502
W160613	0.343808091	0.344545561	W168400	0.335235694	0.419693896
W160704	0.232104369	0.250366102	W168447	0.763660886	0.955009304
W160710	0.396657419	0.427944578	W168500	0.663090208	0.874811375
W160712	0.523502448	0.525848366	W168678	0.484894239	0.48604071
W160718	0.443829434	0.448565431	W168713	0.370150487	0.372763039
W160721	0.340777826	0.346239844	W168729	0.507661663	0.509114628
W160733	0.661954213	0.665382097	W168751	0.507434918	0.546070839
W160783	0.352573618	0.523997868	W168756	0.453319787	0.45523472
W160923	0.184432838	0.184816531	W168787	0.541716664	1.000000000

TABLE B.23: Efficiency scores of stores in the Langeberg region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160176	0.846460524	0.923778672	W166350	1.000000000	1.000000000
W160187	0.33228987	0.416657813	W166396	0.417124375	0.514452237
W160189	0.645856815	0.654993541	W166408	0.549426369	0.592974422
W160191	0.526958463	0.642612418	W166418	0.677254215	0.718716307
W160192	0.726380048	0.794496445	W166419	0.681367363	0.695668326
W160203	0.633376653	0.669937222	W166492	0.586089919	0.674585465
W160205	0.667299781	0.668247018	W166496	0.439244179	0.487714833
W160237	1.000000000	1.000000000	W166505	0.598393323	0.713710144
W160282	0.529002516	0.532839339	W166509	0.342853451	0.418623045
W160283	0.806360458	0.918103728	W166542	0.988320626	0.99363676
W160336	0.599300398	0.640749495	W166563	1.000000000	1.000000000
W160339	1.000000000	1.000000000	W166591	1.000000000	1.000000000
W160350	0.703579266	0.822469146	W166596	0.60490972	0.60490972
W160354	0.743870988	0.871405762	W166599	0.527356218	0.559431977
W160381	0.812188956	1.000000000	W166605	0.952592208	0.995008871
W160419	0.778970607	0.801369574	W166616	0.663298481	0.667518907
W160434	0.737590547	1.000000000	W166631	1.000000000	1.000000000
W160443	1.000000000	1.000000000	W166657	0.78283287	0.83769065
W160445	0.558172158	0.643664674	W166678	0.906667058	0.993179395
W160456	0.843073642	0.856455972	W166687	1.000000000	1.000000000
W160465	0.785739177	0.827001287	W166696	1.000000000	1.000000000
W160476	1.000000000	1.000000000	W168008	0.716890337	0.742073213
W160477	0.52434846	1.000000000	W168031	0.660514425	0.681806839
W160479	0.462588272	0.463418261	W168084	1.000000000	1.000000000
W160533	0.557755258	0.668181747	W168101	1.000000000	1.000000000
W160535	0.536326753	0.605976595	W168123	0.762304952	0.798393968
W160540	0.73236336	0.837301767	W168139	0.515536989	0.562577335
W160541	1.000000000	1.000000000	W168202	0.429811711	1.000000000
W160588	0.482695748	0.698908127	W168213	0.426295921	0.459481258
W160618	0.714116637	0.806481967	W168279	0.730520712	0.741767683
W160629	0.669578781	0.719376971	W168293	0.618339674	0.657531716
W160663	0.826792075	0.894251643	W168297	0.676964095	0.796143719
W160774	1.000000000	1.000000000	W168354	0.759445629	0.793060028
W160975	0.592746117	1.000000000	W168380	0.515525717	0.683430446
W164196	0.810529816	0.886894483	W168488	0.462649289	0.490599937
W164226	0.761199962	0.761649876	W168497	0.409432137	1.000000000
W164266	0.831221261	0.853944968	W168541	0.344510838	0.359182374
W164271	0.812909367	0.814640757	W168597	0.445960134	0.495851119
W166116	0.693587518	0.735795648	W168656	0.633930129	0.655806682
W166137	0.880998244	0.944899785	W168672	0.521695004	0.56875007
W166145	0.545846127	0.682371317	W168703	0.49194995	0.62960219
W166158	0.711416531	0.942069438	W168741	0.530458026	0.535407555
W166159	0.689782283	0.751742885	W168744	0.506807226	0.514603323
W166337	0.630892212	0.638762874	W168791	0.76599737	0.81862281

TABLE B.24: Efficiency scores of stores in the North West region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160108	0.975319501	1.000000000	W164257	0.40273886	0.415303048
W160124	0.128630738	1.000000000	W166118	0.581329855	0.588621744
W160126	0.730838645	0.739668254	W166124	0.634886743	1.000000000
W160134	0.546353658	0.572724297	W166134	0.560742202	0.564920522
W160136	0.890143644	0.977249846	W166171	0.38139887	0.404200191
W160142	0.422965393	0.443352379	W166192	0.43578159	0.456389823
W160155	0.824136086	1.000000000	W166225	0.395200392	0.39626223
W160169	0.73568611	0.773644394	W166299	0.774347023	0.776837592
W160174	0.288419304	0.292789344	W166303	0.302745415	0.318574079
W160195	0.419336847	0.424149168	W166342	0.980570318	1.000000000
W160227	0.750700337	0.769500729	W166389	0.547019142	0.600498053
W160235	0.870121573	0.993574873	W166416	0.376586728	0.376935958
W160241	0.694195919	0.762766123	W166441	0.378903129	0.416617663
W160254	0.837331046	0.85394471	W166473	0.251882459	0.341285051
W160280	0.333654288	0.339962021	W166511	0.606861626	0.632573857
W160313	0.630263196	0.671136326	W166523	0.615239251	1.000000000
W160314	0.55048461	0.662196129	W166547	0.194316556	0.205813384
W160316	0.44667711	0.461709703	W166594	0.501984869	0.524715642
W160322	0.591691927	0.645407133	W166635	0.322812287	0.377081065
W160324	1.000000000	1.000000000	W166660	1.000000000	1.000000000
W160325	0.725255921	0.750119375	W166664	0.509131198	0.539037194
W160347	1.000000000	1.000000000	W166681	0.985535541	0.986575499
W160357	0.697770094	0.708378245	W168045	0.632171448	0.835533357
W160367	0.656236116	0.678656524	W168056	0.683323267	0.776955935
W160387	0.583188653	0.669233191	W168081	0.356703296	0.377126123
W160395	0.610865665	0.697335505	W168109	0.58999434	0.605050127
W160397	0.551862916	0.560733944	W168113	0.467222551	0.490302042
W160421	0.378460285	0.393335678	W168224	0.481422477	1.000000000
W160422	0.575449299	0.614776654	W168332	0.369548343	0.371817892
W160424	0.628791376	0.776761032	W168333	1.000000000	1.000000000
W160444	0.44671737	0.597979048	W168367	0.701451105	1.000000000
W160469	0.698130026	1.000000000	W168385	0.295273263	0.432699866
W160487	0.647836448	0.736949599	W168404	0.36976024	0.870496424
W160489	0.358090657	0.386601232	W168424	1.000000000	1.000000000
W160509	0.457905569	0.460317252	W168454	0.525425057	1.000000000
W160512	0.428814867	0.428956092	W168461	0.747935045	0.752141598
W160525	0.379623977	0.398110379	W168603	0.744972027	0.78161265
W160561	0.442436917	0.443893375	W168611	0.462710632	0.495523878
W160572	0.516435989	0.543472204	W168648	0.749917392	1.000000000
W160614	0.395747049	0.416882072	W168654	1.000000000	1.000000000
W160628	0.455161756	0.48186741	W168690	0.595332146	0.89679795
W160648	0.452717976	0.456189604	W168753	0.387382627	0.392404782
W160690	0.60432249	0.612235128	W168760	0.315731999	0.331019292
W160909	0.441589197	0.71182349	W168768	0.808137391	0.923231124
W160922	0.07140229	0.077873375	W168781	0.606579313	0.744524409
W160965	0.50334963	0.507680446			

TABLE B.25: Efficiency scores of stores in the Free State region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160181	0.474410867	0.474651189	W166462	0.607547386	0.619473012
W160208	0.843611762	1.000000000	W166467	0.639808213	0.679066336
W160210	1.000000000	1.000000000	W166468	1.000000000	1.000000000
W160217	1.000000000	1.000000000	W166494	0.629115334	0.702820841
W160220	0.35155191	0.385943638	W166520	0.413112089	0.42038887
W160222	0.468488722	0.488992396	W166531	0.779059366	0.781648308
W160233	0.426802923	0.430199848	W166533	0.850413929	0.851363536
W160259	0.665854225	0.888166688	W166545	1.000000000	1.000000000
W160285	0.534436109	0.665360533	W166555	0.468579373	0.503883954
W160296	0.562197559	0.833353344	W166564	0.814441608	0.831451974
W160304	0.73396964	0.828691779	W166580	0.493449443	0.523903396
W160311	0.373084181	0.373084181	W166601	0.449835113	1.000000000
W160321	0.679864441	0.755007432	W166607	0.909555684	0.910166224
W160349	0.626670602	1.000000000	W166608	1.000000000	1.000000000
W160414	0.321572909	0.326461564	W166612	0.739200185	0.858291736
W160437	0.416545605	0.481892936	W166630	0.540736413	0.592376622
W160464	0.727193618	0.886650119	W166667	0.861398759	1.000000000
W160474	0.737358432	0.775162534	W166668	0.738544596	0.988763226
W160482	0.649423944	0.664290613	W166684	1.000000000	1.000000000
W160511	0.691074504	0.717999505	W166686	0.931571857	0.950158533
W160546	1.000000000	1.000000000	W166700	1.000000000	1.000000000
W160583	0.720363899	0.759725759	W166701	1.000000000	1.000000000
W160605	0.317139971	0.329564973	W166710	0.098653979	1.000000000
W160643	0.982785179	1.000000000	W168026	0.742964874	0.861454263
W160662	0.413491086	0.426893839	W168063	0.936812594	0.96745596
W160688	0.562776706	0.575387968	W168073	0.463631739	0.466167295
W160772	0.679445149	0.746080977	W168099	0.327149916	0.356457391
W160971	0.995906851	1.000000000	W168107	0.477963145	0.49069254
W160976	0.82559305	0.826557473	W168225	0.468739051	0.47353593
W164170	0.363968592	0.412419901	W168281	0.559298525	0.97901912
W164256	0.661921169	0.684621512	W168359	0.717945683	0.786574311
W164259	0.723909676	0.922255371	W168375	0.503188186	0.512108825
W164527	0.603016718	0.603150501	W168401	0.692070964	0.865269573
W166165	0.477021681	0.587239483	W168495	0.342402916	0.433567068
W166211	0.940145326	1.000000000	W168509	0.638549919	0.731068548
W166236	1.000000000	1.000000000	W168534	0.493765832	0.500237208
W166240	0.609573741	1.000000000	W168573	1.000000000	1.000000000
W166302	0.382415662	0.399740288	W168649	0.254065459	0.286379397
W166334	0.545614304	0.564836371	W168694	0.424820643	0.427420428
W166357	0.683455255	1.000000000	W168712	0.852941739	0.906570098
W166369	0.268041366	0.335667811	W168722	0.340934589	0.342973774
W166409	0.771177544	0.920442785	W168743	0.525971998	0.689111879
W166415	0.851968715	0.862830799	W168773	0.683133811	0.687091252
W166431	0.498872474	0.577340155	W168782	0.939819285	1.000000000
W166449	0.68781753	0.696291309	W168794	0.643284062	0.72175639
W166450	0.469699304	0.474295232			

TABLE B.26: Efficiency scores of stores in the Lesedi region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160137	0.599368692	0.609582628	W166464	0.433359144	0.43954761
W160224	0.418569038	0.487787505	W166472	0.754723305	0.817474836
W160253	0.491488737	0.523103414	W166481	0.424560284	0.454136645
W160352	0.391461191	0.408874766	W166493	0.301667461	0.303555135
W160353	0.585622743	0.607620298	W166500	0.552896589	0.573938514
W160356	0.582510435	0.642254153	W166504	0.369311456	0.58233731
W160452	0.688247134	0.694310101	W166508	0.554988975	0.558003186
W160473	0.383916468	0.384624196	W166513	1.000000000	1.000000000
W160495	1.000000000	1.000000000	W166521	0.381519594	0.383545417
W160501	0.324290494	1.000000000	W166535	0.529633231	0.656712304
W160548	0.442262111	0.564441512	W166537	0.789456915	1.000000000
W160616	0.385065941	0.387848102	W166579	0.279897515	0.287778379
W160651	0.712551408	0.819923254	W166581	0.446275898	0.532848435
W160787	0.604972073	0.702981482	W166586	0.973426591	1.000000000
W160943	0.749106828	0.805136672	W166593	0.50777454	0.508718148
W160949	0.458083782	0.602061219	W166663	0.460077387	0.862706
W160960	0.477706441	0.51280558	W166676	0.439029996	0.635584266
W160987	0.459021991	0.5104027	W166689	0.68330108	0.750750754
W160996	0.511662567	0.576802371	W166690	1.000000000	1.000000000
W166117	0.604480138	0.60785771	W166702	0.855852592	0.861061006
W166120	0.529480932	0.541995599	W168086	0.50314841	0.523881779
W166132	0.302776247	0.35211377	W168211	0.271179699	0.294635684
W166133	0.422466871	0.4257364	W168215	1.000000000	1.000000000
W166140	0.614993335	0.663247576	W168217	0.544578684	0.553410208
W166167	1.000000000	1.000000000	W168235	0.337129942	0.338883543
W166177	0.578850832	0.609647414	W168266	0.214645401	1.000000000
W166182	0.328382533	0.427420767	W168280	0.523416595	0.928061439
W166200	0.343601223	0.346090304	W168376	0.404999161	0.407207434
W166206	0.786307527	0.793552672	W168408	1.000000000	1.000000000
W166213	0.536982102	0.564207292	W168432	0.766510524	1.000000000
W166214	1.000000000	1.000000000	W168460	0.452522766	0.661151684
W166218	0.451366356	0.460199102	W168486	0.632007683	0.678824192
W166219	0.521877803	0.689907255	W168490	0.455964769	0.471454882
W166272	0.916483119	1.000000000	W168502	0.527092184	0.547984008
W166281	0.397729254	0.448421635	W168514	0.278559401	0.369061936
W166320	0.798192785	0.860024351	W168559	0.343900079	0.345002292
W166340	1.000000000	1.000000000	W168574	0.875067083	1.000000000
W166346	1.000000000	1.000000000	W168586	0.638687907	0.764505906
W166352	0.56929705	0.643917397	W168652	0.572826657	1.000000000
W166353	0.342062361	0.499504428	W168660	0.886168544	1.000000000
W166356	0.510981444	0.528306897	W168677	0.972249439	1.000000000
W166395	0.476212691	0.858664524	W168695	0.605783104	0.610201345
W166400	0.43009736	0.433195018	W168724	0.486504635	0.550319536
W166401	0.351752633	0.366286035	W168727	0.826295626	0.892506514
W166405	0.47333192	0.50076801	W168728	0.36385804	0.36653973
W166424	0.580298484	0.989292745	W168758	0.658986598	0.670652536
W166442	0.755877286	0.85203208	W168775	0.330893584	1.000000000
W166451	0.566371704	0.571954262	W168778	0.450377986	0.451923261
W166452	0.449713561	0.453148797	W168793	0.931006857	1.000000000

TABLE B.27: Efficiency scores of stores in the Gauteng region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160132	0.769771709	0.814142858	W166439	0.493147661	0.526539117
W160164	0.683352537	0.701367371	W166445	1.000000000	1.000000000
W160201	1.000000000	1.000000000	W166459	0.236254632	0.244568417
W160211	0.49093359	0.504228463	W166484	0.525585716	0.539781099
W160218	0.374897939	0.382157166	W166488	0.582770729	0.609297386
W160249	0.550823854	0.573802543	W166510	0.527956893	0.545518763
W160272	0.431431992	0.437112199	W166515	0.562501154	0.605074817
W160310	0.436087581	0.448346875	W166518	0.415203529	0.421990445
W160318	0.292061996	0.302432546	W166527	0.36855454	0.372089695
W160338	1.000000000	1.000000000	W166538	0.680293286	0.694267725
W160345	0.688477154	0.734332878	W166544	0.569990132	0.635786394
W160401	0.759673051	0.785296958	W166568	0.121975634	0.149162525
W160425	0.606988098	0.660185406	W166573	0.404518246	0.411882211
W160440	0.412049803	0.423186929	W166617	0.288524449	0.311730184
W160503	0.59123328	0.666938275	W166627	0.531608791	0.550004107
W160582	0.562646685	0.563468411	W166633	0.449979069	0.47548188
W160603	0.360878938	0.373034735	W166639	1.000000000	1.000000000
W160607	0.623816482	0.626364554	W166651	0.48112541	0.54144885
W160620	0.475897029	0.484782292	W166658	0.384624826	0.43313174
W160642	0.30944274	0.319422038	W166698	1.000000000	1.000000000
W160681	0.378600182	0.393733006	W166704	0.368371731	0.391158882
W160696	0.381383169	0.390857538	W168032	0.311558425	0.316177936
W160914	0.525909615	0.586905281	W168035	0.518680828	0.522881836
W160918	0.463476097	0.578501054	W168095	0.504904804	0.509985685
W160919	0.493312678	0.493315073	W168102	1.000000000	1.000000000
W160931	0.518113073	0.540226783	W168103	0.545507452	0.57089549
W160935	0.499791638	1.000000000	W168104	0.59107311	0.61415206
W160967	0.614130394	0.624001761	W168105	0.381207585	1.000000000
W160993	0.350035103	0.382717231	W168212	0.282150395	0.290783236
W164230	0.198081446	0.209725306	W168251	0.325523249	0.364988129
W164258	0.259331119	0.271327008	W168317	0.42370739	0.436247515
W164509	0.149420847	0.178830395	W168326	0.787011474	0.845546027
W164525	0.61339966	0.65449275	W168338	0.481143859	0.489449384
W166101	0.364862798	0.369252407	W168387	0.267971111	0.274641673
W166178	0.396368475	0.412398	W168417	0.337882519	0.339642006
W166231	0.468729272	0.491853869	W168427	0.289421277	0.31341292
W166232	0.42602508	0.42841668	W168527	0.802309034	0.835193364
W166247	0.40893231	0.451709808	W168535	0.737041739	1.000000000
W166265	0.470719769	0.472697767	W168536	1.000000000	1.000000000
W166266	0.607258499	0.631129113	W168548	0.368991404	0.373815152
W166332	0.585398985	0.612670938	W168549	0.343966219	0.419662693
W166336	0.683379205	0.740711722	W168590	0.772187884	0.814043592
W166368	0.864456923	0.901709476	W168663	0.517205435	0.668071957
W166378	1.000000000	1.000000000	W168664	0.204235192	0.215726137
W166382	0.385389701	0.390486339	W168670	1.000000000	1.000000000
W166385	0.739095135	0.739421997	W168715	0.524478303	0.604048985
W166403	0.733307322	0.773450158	W168737	0.723666321	0.789197318
W166430	0.510482965	0.518936873	W168779	0.614208923	0.625939132
W166437	0.334569193	0.342828776	W168780	0.666583351	0.702823083

TABLE B.28: Efficiency scores of stores in the Limpopo region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160141	0.123554542	0.128294581	W166548	0.429798154	0.534761532
W160149	0.312340589	0.320910008	W166549	0.468653333	0.470935441
W160185	0.349947758	0.56103926	W166576	0.295141111	0.436119931
W160276	0.422341941	0.450846739	W166577	0.399477078	0.428049891
W160359	0.229879573	0.232224701	W166598	0.349935346	0.3531688
W160442	0.221685846	0.225617866	W166615	1.000000000	1.000000000
W160458	0.281082076	0.350921551	W166622	0.356934272	0.369754376
W160461	1.000000000	1.000000000	W166632	0.327027168	0.370219836
W160490	0.31781297	0.34583237	W166647	0.617639824	0.682038126
W160532	1.000000000	1.000000000	W166653	0.503355928	1.000000000
W160566	0.378822864	0.426078496	W166655	0.292887199	0.471990575
W160580	1.000000000	1.000000000	W166682	0.699370787	0.70601805
W160604	0.136360819	0.19782178	W168006	0.360765935	0.398598707
W160693	0.586417504	0.730983389	W168011	0.248358479	0.307031653
W160695	0.253578401	0.277311042	W168013	0.3015186	0.382012592
W160735	0.189490254	0.230111456	W168055	0.299860112	0.353591407
W160736	0.278115062	0.312016617	W168238	0.345233187	0.380643034
W160737	0.754086893	0.756214864	W168239	0.171367676	0.234380574
W160740	1.000000000	1.000000000	W168247	0.356153859	0.508228356
W160742	0.250497312	0.363590067	W168261	0.221212979	0.281048016
W160743	0.365336124	0.371747397	W168262	0.322278459	0.334099965
W160746	0.103385811	1.000000000	W168273	0.263240059	0.281052096
W160748	0.43542287	0.51429845	W168285	0.255825371	0.425285261
W160779	0.259630048	0.270618806	W168289	0.150658586	0.332094553
W160785	0.248613879	0.306427862	W168291	0.224681463	0.289499438
W160925	0.295832803	0.536394598	W168303	0.246916669	0.289634348
W160941	0.586263648	0.779567344	W168321	0.348826444	0.745581877
W164278	0.329386314	0.336849932	W168337	0.647717202	0.768069506
W164280	0.202026174	0.255413024	W168352	0.176590575	0.189964976
W166130	0.348639792	0.443462019	W168353	0.347339126	0.383240484
W166157	0.226522173	0.229774107	W168356	0.072994699	0.101047094
W166164	0.1998271	0.251701121	W168425	0.389392575	0.528318931
W166181	0.378356984	0.470326609	W168426	0.294792779	0.298258303
W166189	0.168532048	0.350633062	W168449	0.3220272	0.439150627
W166202	0.554830914	0.639440318	W168462	0.221871006	0.299414131
W166203	0.267714436	0.275814536	W168463	0.332287428	0.464452382
W166217	0.295044999	0.376527285	W168465	0.498183191	0.655236444
W166226	0.355912741	0.464766179	W168477	0.606811936	1.000000000
W166230	0.168967946	0.215080881	W168522	0.334445226	0.378290306
W166244	0.518447444	0.542038263	W168525	0.340025084	0.467039941
W166301	0.25064492	0.308951947	W168545	0.422668279	0.468819704
W166328	0.113282306	0.190748597	W168587	1.000000000	1.000000000
W166344	0.223704311	0.224908378	W168666	0.246008347	0.412965694
W166379	0.445212652	0.448568541	W168682	0.198748322	0.201357444
W166383	0.289079133	0.390331603	W168685	0.112940439	0.142833923
W166387	0.497729355	0.674960495	W168688	0.327755783	0.35567943
W166407	0.146360583	0.223213329	W168692	0.112733448	0.119536993
W166434	0.24152463	0.262445969	W168736	0.240987086	0.294937162
W166457	0.355231388	0.389841519	W168746	0.333939042	0.376438325
W166476	0.689154365	1.000000000	W168766	0.27756479	0.315672029
W166501	0.680271722	0.922885572	W168776	0.199975605	0.307882157
W166529	0.3567301	0.526249997	W168784	0.189531327	0.192064183
W166539	0.215117143	0.239753743			

TABLE B.29: Efficiency scores of stores in the Thekwini region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
W160178	0.501824696	0.501885204	W166522	0.44190971	0.443292322
W160229	1.000000000	1.000000000	W166530	0.343560722	0.375358151
W160274	0.550530055	0.550795379	W166536	0.59818486	0.602087679
W160312	0.277855598	0.308856854	W166552	0.37359722	0.459781287
W160364	0.430978962	0.479134285	W166572	0.455134883	0.455476693
W160389	0.337318573	0.374500507	W166585	0.355252627	0.388336805
W160398	1.000000000	1.000000000	W166602	0.456821775	0.746131327
W160408	0.332766589	0.333597747	W166609	0.125523498	0.127192512
W160436	0.783151252	0.783890111	W166613	0.416462282	0.443697643
W160460	0.312797696	0.314952326	W166614	1.000000000	1.000000000
W160483	0.381893472	0.450450584	W166629	0.534016891	0.545443005
W160498	0.654752485	0.66837933	W166634	1.000000000	1.000000000
W160543	0.623550167	0.623550167	W166640	0.620843248	0.633339532
W160619	0.544499109	0.557332918	W166652	0.326713039	0.43647799
W160640	0.303126792	0.309753009	W166659	0.245403463	0.281857492
W160778	0.107485268	0.122729574	W166665	0.31753505	0.318522854
W160792	0.102960714	0.103086311	W166672	0.286044496	0.384367839
W160911	0.115141859	0.115403034	W166673	0.783140904	0.883724862
W160913	0.39135421	0.52833691	W166680	0.917972188	0.933764524
W160936	0.588622568	0.711572069	W166691	0.617711749	0.619693255
W160953	0.476678418	0.477340234	W166692	1.000000000	1.000000000
W164132	0.282680462	0.303473335	W166693	0.698559212	0.881380473
W164208	0.304800152	0.304800152	W166705	0.935627063	1.000000000
W166102	0.319873419	0.417903646	W168024	0.610117729	0.622640963
W166123	0.258931007	0.264152737	W168082	0.367574624	0.484783659
W166139	0.637916335	0.674919255	W168094	0.522183965	0.536705489
W166173	0.213858808	0.28559772	W168096	0.394202554	0.406697142
W166175	0.343324133	0.345086544	W168097	0.799167267	0.800045872
W166176	0.172644414	0.211600599	W168098	0.739022117	0.785733228
W166209	1.000000000	1.000000000	W168128	0.477928164	0.493444979
W166216	0.507349743	0.512031325	W168223	0.31153448	0.33305447
W166224	0.386562336	0.387595546	W168268	0.761737581	0.837181622
W166280	0.440511007	0.46430886	W168287	0.206373765	0.223237337
W166314	0.313196542	0.327943138	W168288	1.000000000	1.000000000
W166323	0.265337697	0.267657862	W168301	0.606494074	0.710752245
W166325	0.553364124	0.556236527	W168319	0.869619562	0.870622662
W166326	0.364287987	0.378456554	W168336	0.311685513	0.332212909
W166327	0.194745253	0.268192031	W168402	1.000000000	1.000000000
W166339	0.245030823	0.317188542	W168409	0.714952199	0.775848554
W166345	0.424174221	0.449885969	W168411	0.529204833	0.536703034
W166381	1.000000000	1.000000000	W168434	0.693163392	0.821857741
W166411	0.563247544	0.587430268	W168494	0.308544178	0.324438381
W166420	0.401221736	0.473397366	W168513	0.65907284	0.67630318
W166423	0.621596921	0.725549618	W168558	0.413692221	0.416412632
W166433	1.000000000	1.000000000	W168588	0.425624781	0.509125774
W166448	0.28914047	0.314144304	W168646	0.554771495	0.565207219
W166463	0.738778701	0.813693981	W168655	1.000000000	1.000000000
W166471	0.34803998	0.350167683	W168697	0.505248711	0.544916078
W166498	0.436265473	0.448728536	W168717	0.19091295	0.209334957
W166499	0.445325204	0.462280493	W168738	0.746757514	0.800811795
W166503	0.691994481	1.000000000	W168783	0.394861688	0.415640003
W166507	0.527831972	0.67076872	W168790	0.675086799	0.676102394
W166514	0.889151407	0.923867942	W168795	0.365726449	0.456747886
W166517	0.431598275	0.465309934			

TABLE B.30: Efficiency scores of stores in the Tugela region of season W16 under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000105	0.806240007	0.830630822	R006485	0.959340426	0.98626066
R000139	0.686757443	0.694788329	R006487	1.000000000	1.000000000
R000197	0.880518276	0.882392144	R006516	0.355176596	0.367826087
R000198	0.757622841	0.770731346	R006541	0.617589036	0.622001998
R000202	0.669048215	0.706302851	R006554	0.851047202	0.884341751
R000231	0.754063218	0.863225882	R006584	0.64352553	0.679733821
R000328	0.69523376	0.715434261	R006638	1.000000000	1.000000000
R000448	0.656784797	0.673997781	R006649	0.89672475	0.908107115
R000478	0.422231442	0.422445516	R006666	0.660804792	0.661271138
R000734	0.781851271	0.816341296	R006674	0.832822503	0.845560325
R000982	1.000000000	1.000000000	R006679	0.574185464	0.581125918
R004168	0.714263306	0.748900075	R006709	1.000000000	1.000000000
R004172	0.734706349	0.75675878	R008263	1.000000000	1.000000000
R004203	0.633844242	0.658396152	R008510	0.927970407	0.930736278
R005612	0.633844242	0.658396152	R008515	0.751724459	0.7599784
R005672	0.633844242	0.658396152	R008531	0.86524822	0.872012361
R006201	1.000000000	1.000000000	R008683	0.880999456	0.903349373
R006286	1.000000000	1.000000000	R008732	1.000000000	1.000000000
R006324	0.869563194	0.885741254	R008752	0.651788228	0.676459851
R006444	0.740513058	0.752229555			

TABLE B.31: Efficiency scores of stores in the Southern Namibia region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000167	1.000000000	1.000000000	R006623	0.604877608	0.783298637
R000219	0.889479008	0.924111402	R006624	0.743473708	0.762496506
R000256	0.810957434	0.811967572	R006625	0.770246663	0.77813234
R000297	0.802645697	0.809777584	R006626	0.763485179	0.778797893
R000323	0.913658264	0.947340829	R006637	0.759611888	0.760036074
R000326	0.641009396	0.655592411	R006656	0.506640692	0.629702618
R000432	0.597606207	0.63038104	R006669	1.000000000	1.000000000
R000447	0.839536211	0.869437243	R006688	0.420916919	0.42700902
R000467	0.526932409	0.545902866	R006708	1.000000000	1.000000000
R000507	1.000000000	1.000000000	R008271	0.655094136	0.699813387
R000981	0.81476551	0.817783212	R008272	0.805539219	1.000000000
R000984	0.596055387	0.848640395	R008298	0.929635672	0.937385569
R005583	0.596055387	0.848640395	R008340	0.967726766	0.973294375
R005627	0.596055387	0.848640395	R008370	0.978799303	0.994821826
R006144	0.908265957	0.922202595	R008382	0.748077898	0.769457941
R006174	0.976640352	1.000000000	R008391	1.000000000	1.000000000
R006229	0.929589131	0.959514713	R008393	0.77723772	0.789535875
R006469	1.000000000	1.000000000	R008412	0.945616926	0.955686462
R006506	0.763001718	0.771946434	R008420	1.000000000	1.000000000
R006519	0.763557175	0.77996638	R008493	1.000000000	1.000000000
R006543	0.88730101	0.927974923	R008499	1.000000000	1.000000000
R006561	0.755089821	0.766285592	R008520	0.96740631	0.972149082
R006571	0.753686423	0.771165815	R008521	0.779612815	0.799073546
R006582	0.894851613	0.901982909	R008529	1.000000000	1.000000000
R006600	0.616128482	0.634419049	R008539	1.000000000	1.000000000
R006620	0.724123528	0.755732414	R008546	0.758969931	0.776993311

TABLE B.32: Efficiency scores of stores in the Northern Namibia region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000115	0.906808228	0.911528948	R000115	0.906808228	0.911528948
R000199	0.916345848	1.000000000	R000199	0.916345848	1.000000000
R000428	1.000000000	1.000000000	R000428	1.000000000	1.000000000
R000520	0.893959679	1.000000000	R000520	0.893959679	1.000000000
R000536	0.916942025	1.000000000	R000536	0.916942025	1.000000000
R000597	1.000000000	1.000000000	R000597	1.000000000	1.000000000
R000634	0.926228216	0.927866069	R000634	0.926228216	0.927866069
R000907	1.000000000	1.000000000	R000907	1.000000000	1.000000000
R000964	1.000000000	1.000000000	R000964	1.000000000	1.000000000
R000978	0.817450252	0.819640429	R000978	0.817450252	0.819640429

TABLE B.33: Efficiency scores of stores in the Swaziland region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000257	0.860201751	0.860201751	R008274	0.847923044	0.847923044
R000309	0.956530206	0.956943263	R008284	0.782656181	0.782712827
R000402	0.735823659	0.735823659	R008310	0.776732313	0.776732313
R000549	0.815423873	0.815423873	R008311	0.92954792	0.92954792
R000551	1.000000000	1.000000000	R008325	0.930238785	0.930238785
R000552	0.806813041	0.806813041	R008348	1.000000000	1.000000000
R000554	0.952819337	0.952819337	R008349	0.944248063	0.944248063
R000555	0.884181992	0.884546713	R008350	0.879831093	0.879831093
R000556	0.92708863	0.92708863	R008373	0.86569533	0.86569533
R000575	0.880511381	0.880511381	R008374	0.880798131	0.880798131
R000905	0.797810758	0.831484632	R008396	0.874656967	0.874656967
R000926	0.995958329	0.997255279	R008397	1.000000000	1.000000000
R000945	0.857948126	0.857948126	R008398	0.999044014	1.000000000
R006105	0.885343163	0.885343163	R008399	0.977659468	0.977659468
R006119	0.837139877	0.843966084	R008414	0.860947513	0.860947513
R006135	0.838857912	0.838857912	R008415	0.849734948	0.849734948
R006208	0.820663729	0.821723509	R008476	1.000000000	1.000000000
R006234	0.968079014	0.968998023	R008501	0.933310354	0.933310354
R006264	0.849221195	0.849221195	R008537	0.980814666	0.980814666
R006295	0.911638673	0.911638673	R008538	0.970666381	0.970666381
R006341	0.911069913	0.911069913	R008569	0.96246235	0.96246235
R006361	0.919514975	0.919625937	R008571	0.715685968	0.721419982
R006435	0.887147623	0.887147623	R008593	0.884313969	0.884898972
R006440	0.834091913	0.834091913	R008600	0.7336074	0.7336074
R006447	0.816333247	0.819660008	R008665	0.697435813	0.697470898
R006456	0.830217207	0.903517407	R008706	0.900131328	0.900131328
R006470	0.940319037	0.940319037	R008730	0.987095876	0.987095876
R006525	0.808313605	0.830507844	R008731	0.598355999	0.598355999
R006551	1.000000000	1.000000000	R008734	0.904893638	0.904893638
R006560	0.838822315	0.838822315	R008747	0.719830241	0.751808354
R006650	0.838822315	0.838822315	R008749	0.864981264	0.864981264
R008087	0.735647727	0.735647727	R008750	0.784457605	0.789501804
R008204	0.944023168	0.944024662	R008786	0.942297089	0.942297089
R008226	0.91215708	0.91215708	R008797	0.887815674	0.887815674
R008227	1.000000000	1.000000000			

TABLE B.34: Efficiency scores of stores in the Botswana region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000104	0.782819123	0.78452977	R000730	0.971131917	1.000000000
R000109	0.633731434	0.657351828	R000732	0.535634884	0.544906154
R000111	0.767778461	0.821245792	R000780	0.585986064	0.608484505
R000144	0.420736694	0.424769387	R000789	0.751767769	0.753592419
R000148	0.805827502	0.837805438	R000970	0.648398909	0.696024595
R000151	0.781610812	0.784792843	R004101	0.493732404	0.493788298
R000154	0.783959266	0.788485778	R004244	0.95254145	0.966542253
R000156	0.50742464	0.50861267	R006136	1.000000000	1.000000000
R000173	0.799481996	0.802279068	R006146	0.846372173	0.846393369
R000184	0.792022115	0.798444466	R006161	0.76917957	0.80952516
R000188	0.866776692	0.899790501	R006170	1.000000000	1.000000000
R000190	0.986241243	1.000000000	R006268	0.818352362	0.830603815
R000206	0.881720261	0.883853106	R006277	0.891822823	0.914224118
R000226	0.472775978	0.522216844	R006279	0.992203217	1.000000000
R000228	0.506077805	0.553270729	R006317	0.869732834	1.000000000
R000234	0.64771786	0.66637494	R006349	0.489426249	0.489426249
R000247	0.540143426	0.71701903	R006351	0.903593432	0.909798918
R000248	0.762876068	0.792793164	R006355	0.929844159	1.000000000
R000251	0.745665679	1.000000000	R006414	0.810419737	0.814633094
R000270	0.911613791	0.913404432	R006455	0.568736997	0.580610204
R000290	0.582652144	0.890227577	R006465	0.922924958	0.941319139
R000333	0.909746501	0.972625975	R006534	0.559889564	0.575074676
R000341	0.224019662	0.241246149	R006575	0.516266528	0.535011972
R000344	0.564207909	0.571993205	R006590	0.6642074	0.677666251
R000361	0.660518261	0.661036599	R006643	0.937517234	1.000000000
R000366	0.459220584	0.502823887	R006670	1.000000000	1.000000000
R000369	0.65345638	0.665203463	R006671	0.43533312	0.435503562
R000372	0.867363156	0.86760448	R008018	0.585597717	0.612036126
R000382	0.88685996	0.888046915	R008042	0.254788814	0.292821664
R000386	0.9098924	0.921727503	R008046	0.838386518	0.847744993
R000391	0.69434216	0.695413099	R008092	0.539808762	1.000000000
R000405	0.698959957	0.809889283	R008133	0.726966241	0.734774152
R000411	0.704836007	0.711446575	R008365	0.710717371	0.714491968
R000417	0.755967807	1.000000000	R008388	0.534154011	0.822066172
R000457	0.554487472	0.557504248	R008419	1.000000000	1.000000000
R000494	1.000000000	1.000000000	R008487	0.942421634	0.944344833
R000505	0.793373229	0.831074079	R008674	0.756550751	0.763530331
R000565	1.000000000	1.000000000	R008679	0.66252674	0.666250414
R000570	0.816195378	0.823063257	R008696	0.368003645	0.41262823
R000621	0.885874621	0.911893169	R008726	0.655821366	0.661295906
R000723	0.655144304	0.665098444	R008735	0.806346167	0.810622967
R000724	0.85147152	0.864247099	R008792	1.000000000	1.000000000
R000727	0.794559157	0.8087205			

TABLE B.35: Efficiency scores of stores in the Cederberg region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000221	1.000000000	1.000000000	R006422	0.78280944	0.783025212
R000255	0.884677719	0.884677719	R006432	0.91483227	0.91496699
R000271	0.93475028	0.937748024	R006460	0.903506625	0.903506625
R000291	0.918688992	0.925269364	R006461	0.711224413	0.722808133
R000303	1.000000000	1.000000000	R006480	0.951361405	0.951361405
R000335	0.645068994	0.645068994	R006490	0.914898023	0.91793799
R000430	0.68221788	0.68221788	R006495	0.557792609	0.558069807
R000542	1.000000000	1.000000000	R006512	0.989345093	0.989740495
R000630	0.648226261	0.648226261	R006562	0.945879797	0.947939063
R000638	0.954268329	0.962365632	R006565	0.795503973	0.795503973
R000639	1.000000000	1.000000000	R006570	0.76793941	0.76793941
R000674	0.753924889	0.753924889	R006578	0.857484711	0.85984879
R000797	0.901656547	0.9023115	R006588	0.712690262	0.712690262
R000924	0.936427777	0.936427777	R006641	0.972138858	0.980292723
R000989	0.814988689	0.816000836	R006642	0.814034396	0.829833156
R004506	1.000000000	1.000000000	R006654	0.882700218	0.886492473
R006115	0.752136406	0.769736098	R006677	1.000000000	1.000000000
R006127	0.616274298	0.616274298	R008079	0.708788331	0.709560221
R006138	0.654994637	0.654994637	R008114	0.800366425	0.801651303
R006180	0.892152601	0.892152601	R008218	0.737799655	0.74277284
R006184	0.935967398	0.935967398	R008220	0.830864086	0.830864086
R006194	1.000000000	1.000000000	R008258	0.940659423	0.940659423
R006205	0.887873511	0.892107757	R008269	0.86489006	0.86489006
R006220	0.88998376	0.890461056	R008278	0.848498223	0.849139777
R006221	0.845491597	0.849287643	R008306	0.901946472	0.901946472
R006243	0.723171302	0.723283126	R008378	0.79581616	0.801775798
R006246	0.701060297	0.70143483	R008403	0.925866187	0.925866187
R006273	0.899495423	0.902535955	R008430	1.000000000	1.000000000
R006274	0.836597171	0.837014484	R008431	0.954529242	0.962181063
R006321	0.494751034	0.494751034	R008437	0.768354379	0.775628131
R006331	0.643099089	0.643099089	R008438	1.000000000	1.000000000
R006338	0.883650081	0.883650081	R008489	0.722015571	0.725182053
R006358	0.922815216	0.937430736	R008517	0.819951338	0.819951338
R006363	0.958602002	0.958602002	R008592	0.810326097	0.812838068
R006364	0.958602002	0.958602002	R008667	0.792858757	0.795510129
R006367	0.930886849	0.930886849	R008668	0.865662622	0.865662622
R006371	0.893652773	0.893652773	R008671	0.799781965	0.799781965
R006376	0.94349145	0.94349145	R008673	0.407407728	0.41712445
R006377	0.959677115	0.960221177	R008687	0.747670773	0.747670773
R006384	0.847129014	0.847220987	R008689	0.699421744	0.706681514
R006386	0.842325312	0.842325312	R008739	0.854058027	0.854058027
R006390	0.826041736	0.826041736	R008767	0.829801466	0.831321324
R006402	0.912316583	0.91244516	R008772	0.842018226	0.842018226

TABLE B.36: Efficiency scores of stores in the Kwena region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000146	0.763966698	0.807141271	R006366	0.855615996	0.879303903
R000180	0.616512075	0.661976206	R006372	0.76783846	0.773101197
R000223	0.601254308	0.616334422	R006397	0.828346783	0.87531686
R000267	0.720260981	0.746973241	R006413	0.747692662	0.749150105
R000279	0.754840115	0.815823371	R006421	0.814407233	0.882223781
R000289	0.85159969	0.85217579	R006428	0.770030648	0.852268333
R000307	0.69691743	0.755175732	R006436	0.630223966	0.660502709
R000373	0.867112594	0.910523614	R006443	0.747554972	0.795391475
R000400	0.817129959	0.858679049	R006466	0.682595964	0.713119366
R000409	0.615893377	0.664887561	R006482	0.715503223	0.736081952
R000413	0.759582267	0.77288759	R006483	0.775828774	0.775953892
R000418	0.690065712	0.721106737	R006502	0.911606216	0.911951905
R000435	0.721767906	1.000000000	R006524	0.516982468	0.543613083
R000484	0.721767906	1.000000000	R006526	0.872759763	0.873589487
R000499	0.803774605	0.804882105	R006546	0.795518853	0.796201862
R000544	0.874005595	0.907695517	R006550	0.781865547	0.865732985
R000591	0.69235048	0.754571348	R006553	0.571619728	0.622269205
R000610	0.921982865	0.923723411	R006557	0.779483335	0.783561944
R000666	0.85618552	0.934282954	R006558	0.887250982	0.887953404
R000689	0.614193433	0.626813485	R006559	0.788754007	0.789015069
R000796	0.802980742	0.826499394	R006569	0.711232379	0.717683554
R000958	0.841373722	0.849656517	R006606	0.838808874	0.84994708
R000995	0.896308309	0.89724848	R006619	0.780345013	0.781197068
R004209	0.846939191	0.847056954	R006683	1.000000000	1.000000000
R004255	0.749324981	0.751395596	R008023	0.547677023	0.583697975
R004512	0.583074026	0.604909605	R008134	0.797043991	0.808127382
R006104	1.000000000	1.000000000	R008295	0.806499083	0.815841675
R006122	0.633847599	0.643920315	R008384	0.871185996	0.871808971
R006126	0.847335611	0.847427183	R008450	1.000000000	1.000000000
R006141	0.640315133	0.683455294	R008467	0.986744661	1.000000000
R006163	0.595922063	0.677146892	R008511	0.78618516	0.799337362
R006222	0.874668018	1.000000000	R008516	0.84461519	0.857417973
R006235	0.814015868	0.814813591	R008551	0.64539345	0.645580956
R006241	0.827443915	0.852306198	R008552	0.871395266	0.873490036
R006245	0.803931625	0.842183379	R008553	0.681038894	0.774998717
R006252	0.857521179	0.905250535	R008591	1.000000000	1.000000000
R006269	0.831025327	0.831817635	R008681	0.80737137	0.815014527
R006284	0.64437224	0.649228041	R008698	0.767862158	0.767880277
R006285	0.546079176	0.562501882	R008705	0.799625354	0.832261783
R006330	1.000000000	1.000000000	R008711	0.86501046	0.876660673
R006333	1.000000000	1.000000000	R008755	0.788930258	0.800598202
R006335	0.650358478	0.679137673	R008759	0.934544901	0.934666211
R006365	0.554620045	0.577147781	R008769	0.917791954	0.944573859

TABLE B.37: Efficiency scores of stores in the Emfuleni region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000102	0.463643942	0.474635151	R000955	0.562860945	0.696724862
R000112	0.879757931	0.916997311	R004119	0.566674932	0.572504887
R000113	0.661422971	0.716922969	R004231	0.651357802	0.769980088
R000119	0.778506327	0.784377449	R006128	0.741690273	0.751801413
R000131	0.321655091	0.326038596	R006143	0.425687889	0.441289892
R000150	0.549386561	0.6392487	R006160	0.555673298	0.584816267
R000152	0.710323658	0.768101473	R006162	0.877311894	0.9796875
R000160	0.277633754	0.293203753	R006188	0.574460846	0.611830584
R000172	1.000000000	1.000000000	R006215	0.723533287	0.748867158
R000179	0.558506435	0.728758509	R006227	0.751585251	0.759586312
R000182	0.593417513	0.619211481	R006248	0.447296909	0.476142322
R000183	0.447090132	0.459451887	R006287	0.353423416	0.540396574
R000194	0.733897682	0.734422691	R006294	0.855460896	0.857831872
R000215	0.708577749	1.000000000	R006308	0.612739038	0.629284694
R000225	0.396132093	0.555385875	R006311	0.40893677	0.487519093
R000236	0.283128562	0.299589598	R006322	0.551828713	0.555195654
R000245	0.940991322	0.944602914	R006360	0.443574461	0.447694026
R000268	0.402909298	0.452512423	R006375	1.000000000	1.000000000
R000288	0.605802291	0.644811632	R006399	0.455466257	0.457292687
R000320	0.786398175	0.807676999	R006406	0.448650396	0.511677912
R000340	0.478898302	0.488052662	R006426	1.000000000	1.000000000
R000371	0.556923668	0.564837385	R006438	0.868187015	0.87795642
R000374	0.426639598	0.429092148	R006458	0.795639027	0.833653555
R000390	0.432749338	0.45559301	R006478	0.336180171	0.338366747
R000392	0.702377832	0.756637949	R006610	0.468182086	0.550018882
R000396	0.417715842	0.45702588	R006645	0.750518779	0.75558036
R000406	0.476895841	0.503691236	R006661	0.474013521	0.474496935
R000492	1.000000000	1.000000000	R008012	0.389514367	0.394060494
R000513	0.361411252	0.384153399	R008059	0.315959049	1.000000000
R000592	0.752258278	0.799031888	R008065	0.62313184	0.667071078
R000594	0.306212126	0.373829762	R008067	0.684185691	0.687642704
R000598	1.000000000	1.000000000	R008111	0.842646064	0.844996639
R000599	0.634397034	0.638346186	R008315	0.766253543	0.775257844
R000601	0.603506558	0.733955144	R008377	0.978963981	1.000000000
R000613	0.562913431	0.675790816	R008400	0.663229094	0.708675606
R000704	0.568209575	0.570621605	R008447	0.823455553	0.881623218
R000710	0.350863226	0.425040009	R008500	0.862882163	0.864827088
R000712	0.249200434	0.249794481	R008678	0.424036653	0.44743236
R000718	0.612146691	0.639029658	R008713	0.638280806	0.639111326
R000721	0.675635545	0.689269829	R008729	0.619161719	0.624417277
R000733	0.702126374	0.707137643	R008751	0.836737227	0.944508521
R000783	0.677054175	0.678471926	R008756	0.802309283	0.802391762
R000923	0.343494256	0.362929145	R008787	0.967561469	1.000000000

TABLE B.38: Efficiency scores of stores in the Langeberg region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000176	0.93038567	0.934058289	R006350	0.87442558	0.922000629
R000187	0.881321028	0.902240542	R006396	0.958047138	0.96134874
R000189	0.830505649	0.852546647	R006408	0.939655176	1.000000000
R000191	0.882625932	0.890482195	R006418	0.780231146	0.798601337
R000192	0.841838274	0.844834008	R006419	0.90783422	0.993499279
R000203	0.833543641	0.859333025	R006492	0.906202986	0.921335926
R000205	0.922991645	0.932767556	R006496	0.953499355	1.000000000
R000237	0.91286928	0.925707482	R006505	0.775141049	0.790658416
R000282	0.698465717	0.803222478	R006509	0.781880224	0.821043593
R000283	0.994160867	1.000000000	R006542	0.958244706	0.973031835
R000336	0.858582245	0.920076719	R006563	0.792235049	0.818120574
R000339	0.90852385	0.966984135	R006591	0.874268566	0.879178185
R000350	0.833015769	0.844708442	R006596	0.796065335	0.811484635
R000354	0.800791471	0.809978824	R006599	0.761122715	0.765635671
R000381	0.843081657	0.84336378	R006605	0.892308009	0.894028997
R000419	0.966038949	0.978813922	R006616	0.81068479	0.840497725
R000434	0.916477866	0.938413902	R006631	0.818118042	0.830820752
R000443	0.860980747	0.879846305	R006657	0.859610127	0.92576127
R000445	0.75040337	0.767605088	R006678	0.699454226	0.790192382
R000456	0.880408158	0.916078142	R006687	1.000000000	1.000000000
R000465	0.865117989	0.976334293	R006696	1.000000000	1.000000000
R000476	0.821701217	0.829486762	R008008	0.877345036	0.885968387
R000477	0.757958875	1.000000000	R008031	0.779486933	0.780890071
R000479	0.825917842	0.845199796	R008084	0.776866735	0.779283783
R000533	0.902800637	0.9345022	R008101	0.810574293	0.833203021
R000535	0.86932681	0.900014992	R008123	1.000000000	1.000000000
R000540	0.962963577	0.998346197	R008139	1.000000000	1.000000000
R000541	0.959379733	1.000000000	R008202	0.686263157	0.690444809
R000588	0.822984745	0.859733298	R008213	0.912985662	0.970698548
R000618	0.8237497	0.830475238	R008279	0.831483028	0.843379354
R000629	0.891110175	0.905157132	R008293	0.887142981	0.919086308
R000663	0.86238554	0.885268409	R008297	0.930189373	0.98371119
R000774	1.000000000	1.000000000	R008354	0.864336693	0.864750646
R000975	0.777733412	0.815340938	R008380	0.840859552	0.860159358
R004196	0.995472506	0.997466161	R008488	0.776614744	0.78199576
R004226	0.838480561	0.880761809	R008497	0.894631321	0.94703799
R004266	0.898222078	0.903880687	R008541	0.899103418	0.90221845
R004271	0.921346848	0.966539998	R008597	0.832751028	0.847141932
R006116	0.710445355	0.710805865	R008656	0.769877423	0.845041285
R006137	0.889969924	0.890870241	R008672	0.693071253	0.918360317
R006145	1.000000000	1.000000000	R008703	0.842140665	0.896554629
R006158	0.859166635	0.918982337	R008741	0.809913631	0.811511085
R006159	0.755462704	0.764110089	R008744	0.883166607	0.909653835
R006337	0.823610099	0.848784707	R008791	0.661278224	0.665675889

TABLE B.39: Efficiency scores of stores in the North West region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000108	0.69947206	0.71620176	R004257	0.918667596	0.925322779
R000124	0.792472667	0.847278434	R006118	0.86804821	0.876462895
R000126	0.844167727	0.853065207	R006124	0.911247352	0.914076179
R000134	0.739050854	0.769812166	R006134	0.653680346	0.659783132
R000136	0.826434097	0.859188047	R006171	0.748165979	0.766959007
R000142	0.676261681	0.924528962	R006192	0.964477503	0.981353164
R000155	1.000000000	1.000000000	R006225	0.828640089	0.858675534
R000169	0.642815074	0.653023426	R006299	0.909087219	0.911807728
R000174	1.000000000	1.000000000	R006303	0.508269149	0.52041022
R000195	0.691260878	0.693969235	R006342	0.904882431	0.917403825
R000227	1.000000000	1.000000000	R006389	0.827678213	0.829079059
R000235	1.000000000	1.000000000	R006416	0.799225577	0.7997147
R000241	0.794292448	0.79885294	R006441	0.748935896	0.756066056
R000254	0.89971603	0.903752012	R006473	0.6200655	1.000000000
R000280	0.91894824	0.933738805	R006511	0.770370686	0.795270671
R000313	0.693335311	0.699366528	R006523	0.672887956	0.720358428
R000314	0.952419642	0.96582593	R006547	0.67203781	0.67681644
R000316	0.758475565	0.779647195	R006594	0.945028099	1.000000000
R000322	0.72217978	0.726892245	R006635	0.565162372	0.579043458
R000324	1.000000000	1.000000000	R006660	1.000000000	1.000000000
R000325	0.835445218	0.845816289	R006664	0.652856606	0.667601944
R000347	0.62310096	0.689421347	R006681	1.000000000	1.000000000
R000357	0.878466008	0.883615093	R008045	0.654513249	0.686311565
R000367	0.865562206	1.000000000	R008056	0.640010291	1.000000000
R000387	0.963847256	0.97954031	R008081	0.772840419	0.778851718
R000395	0.90594744	0.994305526	R008109	0.855461922	0.873693334
R000397	0.938327827	0.950953739	R008113	0.692102717	0.692242875
R000421	0.66371784	0.668503145	R008224	0.522615964	0.568260397
R000422	0.836518818	0.840959411	R008332	0.951651642	1.000000000
R000424	0.760339601	0.797652635	R008333	0.908429239	1.000000000
R000444	0.862238113	0.882756177	R008367	0.780968395	0.78439442
R000469	0.743025055	0.750442744	R008385	0.821931731	0.8834878
R000487	0.768457715	0.8285144	R008404	0.43393212	0.50131942
R000489	0.887890129	0.902723552	R008424	0.978163858	0.990190319
R000509	0.751466233	0.75436719	R008454	0.990113269	1.000000000
R000512	0.795984587	0.803852638	R008461	1.000000000	1.000000000
R000525	0.859024027	0.911350178	R008603	0.732387476	0.736981637
R000561	0.842340341	0.865289474	R008611	0.817173179	0.825476415
R000572	0.855514179	0.891154027	R008648	0.955945501	0.961990827
R000614	0.986977521	1.000000000	R008654	0.820596879	0.820863768
R000628	0.732919137	1.000000000	R008690	0.955271675	0.972740769
R000648	0.730308336	0.736531809	R008753	0.98792465	1.000000000
R000690	0.864681987	0.884776496	R008760	0.716133892	0.782039927
R000909	0.781884484	0.783348453	R008768	0.76794531	0.814602947
R000922	0.714471253	0.729513518	R008781	0.924277124	0.93446066
R000965	0.855751112	0.858322506			

TABLE B.40: Efficiency scores of stores in the Free State region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000181	0.832949477	0.851925859	R006462	0.860805862	0.904178463
R000208	0.826078347	0.838094094	R006467	0.795446065	0.83579367
R000210	1.000000000	1.000000000	R006468	0.796365041	0.796511938
R000217	0.79261974	0.79841797	R006494	0.786100366	1.000000000
R000220	0.953046421	0.976232994	R006520	0.833343059	0.835385929
R000222	0.691629011	0.702341203	R006531	0.831706298	0.841214423
R000233	0.582892676	0.607238837	R006533	0.946589658	0.950369062
R000259	0.551096598	0.65757421	R006545	1.000000000	1.000000000
R000285	0.464076239	0.573260756	R006555	0.808115218	0.84819055
R000296	0.745274877	0.768963411	R006564	0.765334833	0.767182256
R000304	0.917451598	0.92025336	R006580	1.000000000	1.000000000
R000311	0.886880614	0.952480649	R006601	0.911629963	1.000000000
R000321	0.789311627	0.796516882	R006607	0.832846695	0.846349198
R000349	0.878621146	0.892775127	R006608	0.867259313	0.875330568
R000414	0.657796759	0.718865231	R006612	0.904729737	0.916450728
R000437	0.842522964	1.000000000	R006630	0.958238191	0.981473704
R000464	0.876284739	0.90321542	R006667	0.88766531	0.934169763
R000474	0.782782476	0.855577119	R006668	0.487376149	0.524941571
R000482	0.992290305	0.992368985	R006684	0.821267307	1.000000000
R000511	0.961519948	0.999930194	R006686	0.969762685	1.000000000
R000546	0.794111692	0.961411558	R006700	1.000000000	1.000000000
R000583	0.631374398	1.000000000	R006701	0.659391767	0.681927056
R000605	0.828103471	0.936621102	R006710	1.000000000	1.000000000
R000643	0.884850914	0.892238011	R008026	0.683022828	0.696826998
R000662	1.000000000	1.000000000	R008063	1.000000000	1.000000000
R000688	0.959284337	0.967184614	R008073	0.814536329	0.821265272
R000772	0.799516316	0.799546294	R008099	0.810331744	0.869956304
R000971	0.839877757	0.862019334	R008107	0.822251022	0.831862605
R000976	1.000000000	1.000000000	R008225	0.894310759	0.917792647
R004170	0.7230546	0.725149646	R008281	0.843943458	0.853149513
R004256	0.960014443	0.976614694	R008359	0.962206911	1.000000000
R004259	0.811151934	0.954378081	R008375	0.832301794	0.843175078
R004527	0.669638611	0.690971012	R008401	0.725745935	0.75426957
R006165	0.783247222	0.791198132	R008495	0.66897363	0.671103865
R006211	0.740368629	0.749009595	R008509	0.795234878	1.000000000
R006236	0.802166981	0.804750269	R008534	0.799365634	0.813317474
R006240	1.000000000	1.000000000	R008573	0.881113597	0.885307178
R006302	0.664638384	0.679053005	R008649	0.533687734	0.566356284
R006334	0.878164133	0.886053652	R008694	0.845834781	0.971108608
R006357	0.699536448	0.720105122	R008712	0.571626465	0.585763177
R006369	0.540920192	0.545669339	R008722	0.83288604	0.834299539
R006409	0.911427095	0.967471304	R008743	0.858399918	0.86029997
R006415	0.820312591	0.855212228	R008773	0.73509175	0.764818898
R006431	0.684747765	0.707370205	R008782	0.638115744	0.67971628
R006449	0.699741443	0.910390376	R008794	0.863142979	0.870834535
R006450	0.866425858	0.951119745			

TABLE B.41: Efficiency scores of stores in the Lesedi region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000137	0.679280256	0.778744752	R006464	0.847318252	0.847380793
R000224	0.772754254	0.773753791	R006472	0.944516995	0.948992616
R000253	0.823598371	1.000000000	R006481	0.803972154	0.804215953
R000352	1.000000000	1.000000000	R006493	0.868103716	0.868109659
R000353	0.750918847	0.751011934	R006500	0.986043848	0.99254032
R000356	0.800234264	0.815099419	R006504	0.825670389	0.838245383
R000452	0.97782229	0.980745053	R006508	0.713923036	0.768283324
R000473	0.660802683	0.665413948	R006513	0.643554524	0.702376961
R000495	0.889744512	1.000000000	R006521	0.894846244	0.896763689
R000501	0.900365969	0.901745574	R006535	0.822927571	0.827862502
R000548	0.756980185	0.888387405	R006537	0.909255528	0.923249529
R000616	1.000000000	1.000000000	R006579	0.735167171	0.737446559
R000651	0.792336387	0.805789903	R006581	0.916659556	0.934799526
R000787	0.599177271	0.602575281	R006586	0.85153924	0.860174024
R000943	0.951002885	0.971702087	R006593	0.773706019	0.796884816
R000949	0.932361166	0.933618599	R006663	1.000000000	1.000000000
R000960	0.756452622	0.758143246	R006676	0.713544532	0.729084447
R000987	0.996125562	1.000000000	R006689	1.000000000	1.000000000
R000996	0.833308255	0.969672785	R006690	1.000000000	1.000000000
R006117	1.000000000	1.000000000	R006702	0.739483543	0.881777278
R006120	0.644604904	0.645566734	R008086	0.757893409	0.816043195
R006132	0.690530461	0.694569802	R008211	0.710340683	0.713048517
R006133	0.676893155	0.700492559	R008215	0.774995972	0.825802929
R006140	0.938508157	0.942282161	R008217	1.000000000	1.000000000
R006167	0.970645326	1.000000000	R008235	0.695840135	0.697415514
R006177	0.87287978	0.875137386	R008266	0.953461254	0.953468615
R006182	0.817953426	0.817982395	R008280	0.88117919	0.890162176
R006200	0.820349781	0.829009389	R008376	1.000000000	1.000000000
R006206	0.666705059	0.712988114	R008408	0.854029332	0.879202966
R006213	0.83714987	0.865645882	R008432	0.324967266	0.338526582
R006214	0.808973798	0.821046687	R008460	0.907806068	0.911083168
R006218	0.882209858	0.883897513	R008486	0.637930157	1.000000000
R006219	0.803849203	0.806204157	R008490	0.912848591	0.937522234
R006272	0.947464081	0.947608275	R008502	0.759296875	0.769060662
R006281	0.864525078	0.869807514	R008514	0.87668946	0.884530964
R006320	0.815469687	0.838853123	R008559	0.869036134	0.869443605
R006340	0.936432224	0.93653557	R008574	1.000000000	1.000000000
R006346	0.829033626	0.84083726	R008586	0.850213036	0.851530858
R006352	0.839964467	0.841150722	R008652	0.806608511	0.808679588
R006353	0.923169286	0.923751143	R008660	1.000000000	1.000000000
R006356	0.914036888	0.914050855	R008677	0.83649656	0.860886598
R006395	0.795593129	0.798461582	R008695	0.650143148	0.732190109
R006400	0.627049782	0.651689236	R008724	0.775786093	0.778370975
R006401	0.905711021	0.907258938	R008727	1.000000000	1.000000000
R006405	0.749253743	0.77875189	R008728	0.468115226	0.509455154
R006424	0.819570375	0.836561264	R008758	0.925570924	1.000000000
R006442	0.932195922	1.000000000	R008775	0.823980073	0.829222482
R006451	0.954577296	0.990819365	R008778	0.943420956	0.98335853
R006452	0.808703777	0.808719914	R008793	0.936525461	0.94354649

TABLE B.42: Efficiency scores of stores in the Gauteng region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000132	0.805931947	1.000000000	R006439	0.867927742	0.883335996
R000164	1.000000000	1.000000000	R006445	0.878195138	0.993710966
R000201	0.804280049	0.848637686	R006459	0.821178045	0.836586989
R000211	1.000000000	1.000000000	R006484	0.920913394	0.920943573
R000218	0.783549643	0.803839897	R006488	0.961535711	0.964918349
R000249	0.839958468	0.856197136	R006510	0.84636885	0.84838208
R000272	0.727006425	0.729984655	R006515	0.968308847	0.969251938
R000310	0.799892419	0.82462277	R006518	0.874037912	0.877571068
R000318	0.839105737	0.848986559	R006527	0.82972942	0.845972521
R000338	1.000000000	1.000000000	R006538	0.902513499	0.905304968
R000345	0.801598829	0.802598654	R006544	0.865093438	0.878786721
R000401	0.835589991	0.835867583	R006568	1.000000000	1.000000000
R000425	0.84554145	0.889064711	R006573	0.701658997	0.740278352
R000440	0.853158114	0.854251613	R006617	0.847898864	0.885282296
R000503	0.893662751	0.931844675	R006627	0.814881821	0.937748299
R000582	0.800419535	0.808455325	R006633	0.662561442	0.663865292
R000603	0.80929824	0.817567854	R006639	0.863452237	0.877466474
R000607	0.927302328	0.942883075	R006651	0.876146698	0.877605538
R000620	0.81067821	0.828094485	R006658	0.914807253	1.000000000
R000642	0.585543533	0.604927515	R006698	0.738745971	1.000000000
R000681	0.796968859	0.837371869	R006704	0.392869001	0.427812397
R000696	0.522058299	0.542464119	R008032	1.000000000	1.000000000
R000914	0.884748864	0.920108664	R008035	0.886614038	0.941745664
R000918	1.000000000	1.000000000	R008095	0.702800929	0.931847514
R000919	0.917436748	0.934774018	R008102	1.000000000	1.000000000
R000931	0.796471156	1.000000000	R008103	0.762222491	0.777154915
R000935	0.843645568	0.862681235	R008104	0.928852381	0.93217732
R000967	0.906247583	0.91777869	R008105	0.920158926	1.000000000
R000993	0.927493496	0.95015489	R008212	0.574251319	0.850136353
R004230	0.814159144	0.961597761	R008251	0.393722585	1.000000000
R004258	0.649218462	0.668035035	R008317	0.851338653	0.86073932
R004509	0.749902662	1.000000000	R008326	0.962438424	0.96635639
R004525	0.901611208	0.914021092	R008338	0.617472476	0.673123667
R006101	0.746549852	0.774889867	R008387	0.916464323	0.953069514
R006178	0.842885388	0.879961188	R008417	0.844583078	1.000000000
R006231	0.906498621	0.91634981	R008427	0.614540668	1.000000000
R006232	0.812901602	0.850909438	R008527	0.936476437	0.949351199
R006247	0.992894136	1.000000000	R008535	0.912224874	0.918038175
R006265	0.835095829	0.85816569	R008536	0.822996304	0.887507987
R006266	0.833058532	0.833183394	R008548	0.932186504	0.934538716
R006332	0.86340214	0.868708923	R008549	0.942398842	0.953000158
R006336	0.844321589	0.858429571	R008590	0.952467108	0.967903302
R006368	0.729052449	0.769296584	R008663	0.881505966	0.882906173
R006378	0.930195721	0.934786782	R008664	0.914491324	0.94158541
R006382	0.759007303	0.834382311	R008670	0.859066704	0.867534766
R006385	0.891413551	0.891830813	R008715	0.888240323	0.897958651
R006403	0.832092978	0.862351175	R008737	1.000000000	1.000000000
R006430	0.900657976	0.913335644	R008779	0.87412872	0.88631213
R006437	0.882635228	0.890217897	R008780	0.902176902	0.925340067

TABLE B.43: Efficiency scores of stores in the Limpopo region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000141	0.814106103	0.825260305	R006548	0.894344218	0.907727325
R000149	0.920879882	0.951815995	R006549	0.916970517	0.981877855
R000185	1.000000000	1.000000000	R006576	0.726411913	0.749153951
R000276	0.904219595	0.934021677	R006577	0.81814157	0.855817618
R000359	0.862112237	0.863368337	R006598	0.819758886	0.86168451
R000442	0.787632715	0.79283534	R006615	0.835018093	0.877657545
R000458	0.90618459	0.942392018	R006622	0.557300613	0.562957873
R000461	0.84058606	0.884627372	R006632	0.787558	0.820667466
R000490	0.785035247	0.806895969	R006647	1.000000000	1.000000000
R000532	0.793596061	0.830899897	R006653	0.82599388	0.861164354
R000566	0.771711835	0.809558169	R006655	0.783252724	0.784438924
R000580	0.97150653	1.000000000	R006682	0.621507408	1.000000000
R000604	0.7700232	0.847700895	R008006	0.827918076	0.82984775
R000693	0.808985645	0.844034219	R008011	0.915378042	0.923058635
R000695	0.866955656	0.899312442	R008013	0.962838948	1.000000000
R000735	0.847800212	0.862712682	R008055	0.890527719	0.911967367
R000736	0.836101941	0.859194691	R008238	0.753683928	0.776149031
R000737	1.000000000	1.000000000	R008239	0.666211135	0.690462454
R000740	0.857699023	0.861564208	R008247	0.741926722	0.796954225
R000742	0.878611796	0.903121302	R008261	0.672710328	0.680287608
R000743	0.943368644	0.943654309	R008262	0.732329394	0.751790408
R000746	0.702533731	0.779651648	R008273	0.9638836	0.986939468
R000748	1.000000000	1.000000000	R008285	0.894650593	0.915632417
R000779	0.872549358	0.874981155	R008289	0.688896834	0.708086872
R000785	0.861217865	0.867375003	R008291	0.727378782	0.79539314
R000925	0.858163188	0.868032524	R008303	0.816977529	0.834868395
R000941	0.714888718	0.749960494	R008321	1.000000000	1.000000000
R004278	0.938474322	1.000000000	R008337	0.598718193	0.627157061
R004280	0.86674146	0.90733744	R008352	0.889008301	0.942783453
R006130	0.714372998	1.000000000	R008353	0.831619063	0.863878159
R006157	0.95386883	0.987261037	R008356	0.821425213	0.852812217
R006164	0.745595635	0.879844894	R008425	0.823721065	0.842664648
R006181	0.732704996	0.740163341	R008426	0.759912962	0.779723466
R006189	0.837808569	0.838587862	R008449	0.87531155	0.899981869
R006202	0.764950409	0.813989981	R008462	0.715465087	0.79967686
R006203	0.929373398	0.963880233	R008463	0.859425596	0.875753776
R006217	0.651597044	0.750897225	R008465	0.763127507	0.779167075
R006226	0.585361019	0.639545725	R008477	1.000000000	1.000000000
R006230	0.706703918	0.716171485	R008522	0.905609427	0.913377035
R006244	0.925286418	0.961501483	R008525	0.972752339	0.989918733
R006301	0.69049265	0.746269995	R008545	0.814113545	0.831468821
R006328	0.844920847	0.874826462	R008587	0.85752591	0.858136577
R006344	0.854261233	1.000000000	R008666	0.831966084	0.857165861
R006379	0.878841331	0.963930415	R008682	0.93573417	1.000000000
R006383	0.836721909	0.841277462	R008685	0.756553877	0.78608678
R006387	0.785114314	0.789294968	R008688	0.949372646	0.952795599
R006407	0.628146623	1.000000000	R008692	0.673789526	0.7394892
R006434	0.687897829	0.727789653	R008736	0.842641083	0.847567118
R006457	0.852308168	0.85391834	R008746	0.757995427	0.792566965
R006476	0.972632286	1.000000000	R008766	0.660633025	0.77090711
R006501	0.792400491	0.819723532	R008776	0.80319383	0.808116154
R006529	1.000000000	1.000000000	R008784	0.867478376	0.870334315
R006539	0.55738791	0.583090916			

TABLE B.44: Efficiency scores of stores in the Thekwini region of replenishment products under CRS and VRS.

Store ID	θ_{CRS}	θ_{VRS}	Store ID	θ_{CRS}	θ_{VRS}
R000178	0.740443322	0.829697623	R006522	0.619228952	0.668987054
R000229	1.000000000	1.000000000	R006530	0.454253366	0.475675774
R000274	0.865484871	0.885451222	R006536	0.792076445	0.811422956
R000312	0.829419708	1.000000000	R006552	0.721145171	0.736621812
R000364	0.5838992	0.602336378	R006572	0.586255448	0.631576046
R000389	0.723645243	0.753136324	R006585	0.647036443	0.686559786
R000398	0.905235641	0.917982447	R006602	0.708233078	0.74460436
R000408	0.830842209	0.880739377	R006609	0.956273646	0.968358598
R000436	0.782442335	0.785855007	R006613	0.641118906	0.655683671
R000460	0.794022977	0.828716622	R006614	0.896865298	0.899276936
R000483	0.790685112	0.841721181	R006629	0.782760812	0.838774472
R000498	0.965852973	1.000000000	R006634	0.79598007	0.800222175
R000543	0.819468652	0.821727987	R006640	0.775835073	0.777368721
R000619	0.839035567	0.978133997	R006652	0.646339914	0.650076822
R000640	0.689055624	0.762264333	R006659	0.806010655	1.000000000
R000778	0.68447299	0.712871735	R006665	0.812122724	0.828083067
R000792	0.707605003	0.71745738	R006672	0.631229962	0.876764728
R000911	0.63429622	0.646541973	R006673	0.742153228	0.755964271
R000913	0.915550799	0.927462871	R006680	0.870299834	0.903341904
R000936	1.000000000	1.000000000	R006691	0.479703855	1.000000000
R000953	0.756003165	0.767196582	R006692	0.695926928	1.000000000
R004132	0.875251945	0.898784376	R006693	0.476200213	0.58023584
R004208	0.656493304	0.677122176	R006705	0.946879046	1.000000000
R006102	0.77685699	0.811451836	R008024	0.858608024	1.000000000
R006123	0.764375685	0.78412921	R008082	0.881870942	0.901507561
R006139	0.889932891	0.895781133	R008094	0.97408308	0.983669924
R006173	0.627944541	0.644143077	R008096	0.782588995	0.930146176
R006175	1.000000000	1.000000000	R008097	0.782548667	0.801349856
R006176	0.4520379	1.000000000	R008098	0.924086417	1.000000000
R006209	0.925582975	0.94826944	R008128	0.61945205	0.644973648
R006216	0.817171115	0.858763416	R008223	0.588577195	0.613926535
R006224	0.925643029	0.966722663	R008268	0.587102971	0.637787592
R006280	0.482717764	0.483164215	R008287	0.7533396	0.761274983
R006314	0.725483229	1.000000000	R008288	1.000000000	1.000000000
R006323	0.814178374	0.847246162	R008301	0.957221372	0.990759975
R006325	0.609137193	0.647024657	R008319	0.571040189	0.572190729
R006326	0.807791998	0.838628603	R008336	0.769551388	0.795687849
R006327	0.618997008	0.705218708	R008402	0.703897231	0.722999941
R006339	0.766857117	0.81183772	R008409	0.807552989	0.808667706
R006345	0.771437949	0.773510395	R008411	0.867804883	0.87934633
R006381	1.000000000	1.000000000	R008434	0.893875342	0.894708334
R006411	0.924050187	1.000000000	R008494	0.68875611	0.728970606
R006420	0.728994326	0.776949085	R008513	0.964950026	1.000000000
R006423	0.633347228	0.635798319	R008558	0.863867836	0.878470433
R006433	0.79398426	0.812804914	R008588	0.889062905	0.904363742
R006448	0.655302201	0.669950804	R008646	0.790961256	0.807163085
R006463	0.680494226	0.68222117	R008655	1.000000000	1.000000000
R006471	0.900269185	0.963064698	R008697	0.623891626	1.000000000
R006498	0.64110149	0.658701328	R008717	0.567577734	0.623388875
R006499	0.77604492	0.787887447	R008738	0.795699725	0.811165909
R006503	0.782784134	0.800624699	R008783	0.498537513	0.543893488
R006507	0.976775759	0.991991402	R008790	0.627202428	0.668623012
R006514	0.575455365	0.580987947	R008795	0.726634592	0.879023175
R006517	0.600249313	1.000000000			

TABLE B.45: Efficiency scores of stores in the Tugela region of replenishment products under CRS and VRS.